Linear Waveguides

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Modes

- permittivity $\epsilon = \epsilon(x, y)$ does not depend on z
- wave vector along z axis
- all fields are of the form
  $F(t, x, y, z) = F(x, y)\,\mathrm{e}^{\mathrm{i}\beta z}\,\mathrm{e}^{-\mathrm{i}\omega t}$
- β is the propagation constant of the mode
- there are two kind of modes:
- quasi TE - Transversal Electric
- quasi TM - Transversal Magnetic
- they have different propagation constants
- no analytic solution

Mode equation

- recall the general mode equation

$$\mathbf{curl}\,\mathbf{curl}\,\boldsymbol{E} = k_0^2\,\epsilon(x,y)\,\boldsymbol{E}$$

- the curl operator is

$$\begin{pmatrix} 0 & -\mathrm{i}\beta & \partial_y \\ \mathrm{i}\beta & 0 & -\partial_x \\ -\partial_y & \partial_x & 0 \end{pmatrix}$$

- apply it twice

$$\begin{pmatrix} \beta^2 - \partial_y^2 & \partial_x\partial_y & \mathrm{i}\beta\partial_x \\ \partial_x\partial_y & \beta^2 - \partial_x^2 & \mathrm{i}\beta\partial_y \\ \mathrm{i}\beta\partial_x & \mathrm{i}\beta\partial_y & -\partial_x^2 - \partial_y^2 \end{pmatrix} \boldsymbol{E} = k_0^2\,\epsilon(x,y)\,\boldsymbol{E}$$

Eliminate E_z

- problem: β and β^2
- problem: two polarization states, three fields
- divergence of $\epsilon\,\boldsymbol{E}$ vanishes
- $-\mathrm{i}\beta E_z = \epsilon^{-1}\partial_x \epsilon E_x + \epsilon^{-1}\partial_y \epsilon E_y$
- now the mode equation contains only two fields

$$\begin{pmatrix} k_0^2\epsilon + \partial_x\epsilon^{-1}\partial_x\epsilon + \partial_y^2 & \partial_x\epsilon^{-1}\partial_y\epsilon - \partial_x\partial_y \\ \partial_y\epsilon^{-1}\partial_x\epsilon - \partial_y\partial_x & k_0^2\epsilon + \partial_x^2 + \partial_y\epsilon^{-1}\partial_y\epsilon \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \beta^2 \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

- and it is a normal eigenvalue problem!
- analogous form for magnetic fields

Quasi TE and TM modes

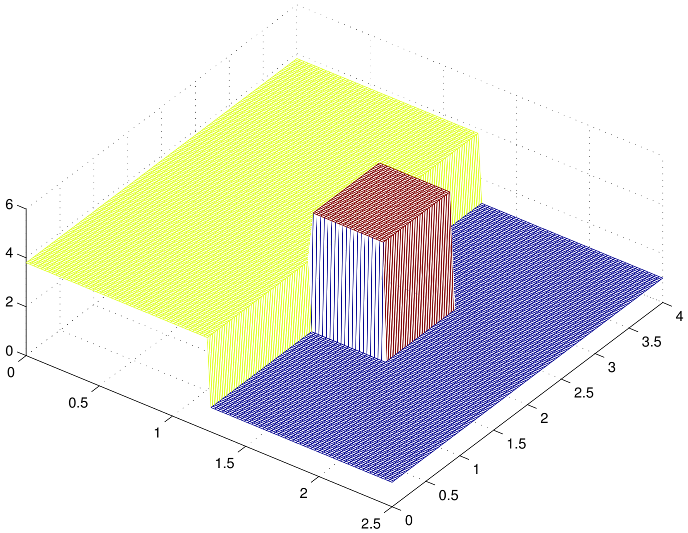
- If waveguides are broad: $\partial_y \epsilon \approx \epsilon \partial_y$
- $E_x \approx 0$
- this results in the quasi TE mode equation
  $\{\partial_x^2 + \partial_y^2 + k_0^2\, \epsilon(x,y)\} E_y = \beta^2\, E_y$
- Helmholtz equation
- with $\partial_y \epsilon \approx \epsilon \partial_y$ and $H_x \approx 0$
- quasi TM mode equation
  $\{\epsilon \partial_x \epsilon^{-1} \partial_x + \partial_y^2 + k_0^2\, \epsilon(x,y)\} H_y = \beta^2 H_y$
- only change is $\epsilon = \epsilon(x,y)$ and additional ∂_y^2
- and: quasi modes have z-components

Hermann von Helmholtz, German physicist, 1821 - 1894

Rib waveguide

- on top of a substrate there is a rib of enhanced permittivity
- cross section represented by $(x_i, y_j) = (ih_x, jh_y)$ with integer numbers i, j
- permittivity $\epsilon(x_i, y_j)$ represented by a matrix `eps`
- define waveguide `rwg`
- digitize it
- setup Helmholtz matrix
- solve for guided modes
- visualize result

```
function rwg=define
clear all;
% all lengths in microns
rwg.LAMBDA=1.30;
rwg.EC=1.00; % cover permittivity
rwg.ES=3.80; % substrate permittivity
rwg.ER=5.80; % rib permittivity
% computational window, [xlo,xhi,ylo,yhi]
rwg.CW=[0.0,2.5,0.0,4.0];
% rib, [xlo,xhi,ylo,yhi]
rwg.RB=[1.25,1.75,1.5,2.5];
end % function define
```



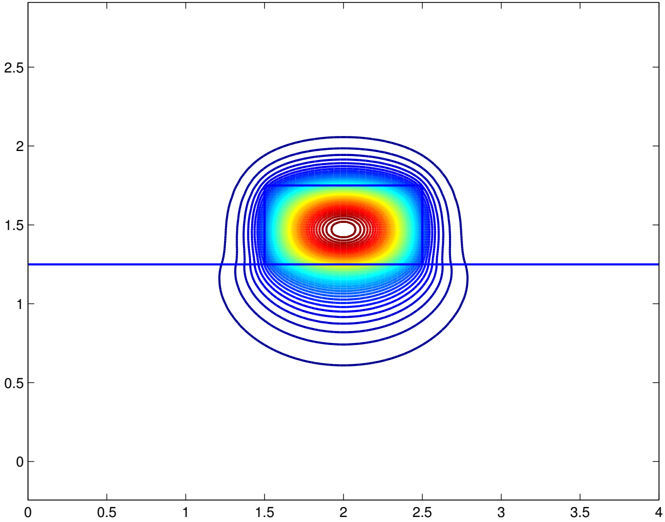
Permittivity profile of a rib wabeguide

```
function rwg=digitize(r,NX,NY)
x=linspace(r.CW(1),r.CW(2),NX);
y=linspace(r.CW(3),r.CW(4),NY);
[X,Y]=meshgrid(x,y);
RIB=(X>r.RB(1))&(X<r.RB(2))&(Y>r.RB(3))&(Y<r.RB(4));
SUB=(X<=r.RB(1));
COV=(X>r.RB(1))&~RIB;
prm=r.EC*COV+r.ES*SUB+r.ER*RIB;
rwg=r;
rwg.x=x;
rwg.y=y;
rwg.eps=prm;
% note that NX and NY may be reconstructed
% from rwg.eps
end % function digitize
```

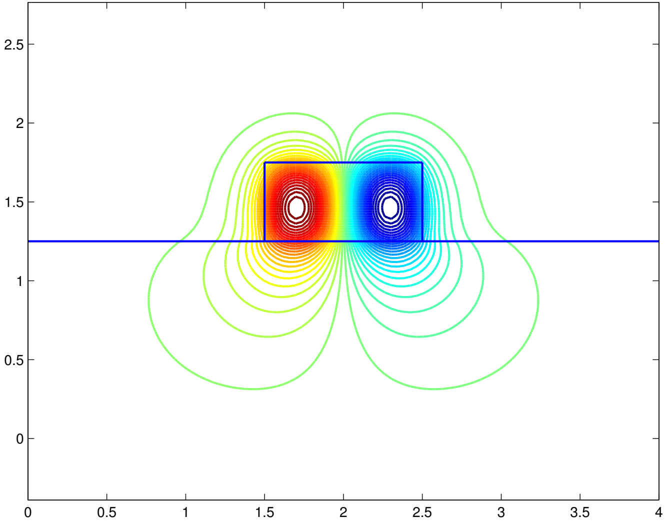
```
function rwg=helmholtz(r)
k0=2*pi/r.LAMBDA;
hx=r.x(2)-r.x(1); % x step width
hy=r.y(2)-r.y(1); % y step width
[NX,NY]=size(r.eps'); % size of computational window
NV=NX*NY; % number of variables
prm=reshape(r.eps',NV,1); % permittivity
one=ones(NV,1);
md=-2*one*(1/hx^2+1/hy^2)/k0^2+prm; % main diagonal
xd=one/hx^2/k0^2; % side diagonal xx differentiation
yd=one/hy^2/k0^2; % side diagonal yy differentiation
hh=spdiags([yd,xd,md,xd,yd],[-NX,-1,0,1,NX],NV,NV);
% note that there are false links, remove
for n=NX:NX:NV-NX
  hh(n,n+1)=0;
  hh(n+1,n)=0;
end;
rwg=r;
rwg.hh=hh;
end % function helmholtz
```

```
function rwg=solve(r)
% calculate 6 eigenvectors of sparse matrix
% algebraically largest eigenvalues
[efun,eval]=eigs(r.hh,6,'la');
eval=diag(eval);
% guided modes
guided=(r.ES<eval);
modes=efun(:,guided);
[NX,NY]=size(r.eps);
rwg=r;
% modes must be reshaped
rwg.mode=reshape(modes,[NY,NX,sum(guided)]);
end % function solve
```

```
function visualize(r,m,fn)
close;
[~,h0]=contour(r.y,r.x,r.mode(:,:,m),48);
hold on;
h1=plot([r.CW(3),r.CW(4)],[r.RB(1),r.RB(1)]);
h2=plot([r.RB(3),r.RB(3)],[r.RB(1),r.RB(2)]);
h3=plot([r.RB(4),r.RB(4)],[r.RB(1),r.RB(2)]);
h4=plot([r.RB(3),r.RB(4)],[r.RB(2),r.RB(2)]);
hold off;
axis equal;
set([h0,h1,h2,h3,h4],'LineWidth',1.5);
if ~strcmp(fn,'')
  epsfn=[fn,'.eps'];
  print('-depsc',epsfn);
  command=['bash ','epstopdf ',epsfn];
  system(command);
  delete(epsfn);
end;
end % function visualize
```



The base mode of a model rib waveguide.



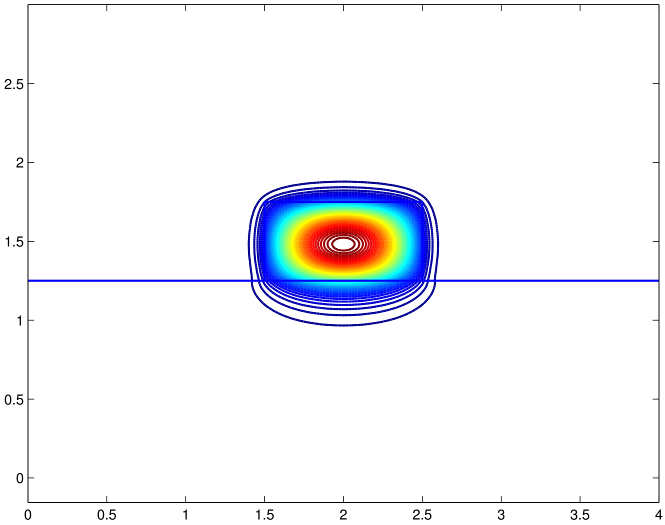
Excited mode. Note that we have plotted the amplitude.

More

- you may edit an existing rib waveguide descriptor `rwg`
- function `ribwg` for the chain `digitize`, `helmholtz` and `solve`
- Matlab programs as `ribwg.zip` at the APS server
- `ftp://202.113.31.42/temp/peter.hertel/2011-03`
- files will be updated when errors have been detected
- please report errors to `phertel@uos.de`

A typical rib waveguide descriptor

```
     LAMBDA: 0.6330
         EC: 1
         ES: 3.8000
         ER: 5.2000
         CW: [0 2.5000 0 4]
         RB: [1.2500 1.7500 1.5000 2.5000]
          x: [1x200 double]
          y: [1x200 double]
        eps: [200x200 double]
         hh: [40000x40000 double]
       mode: [200x200x5 double]
```



Its base mode, TE0

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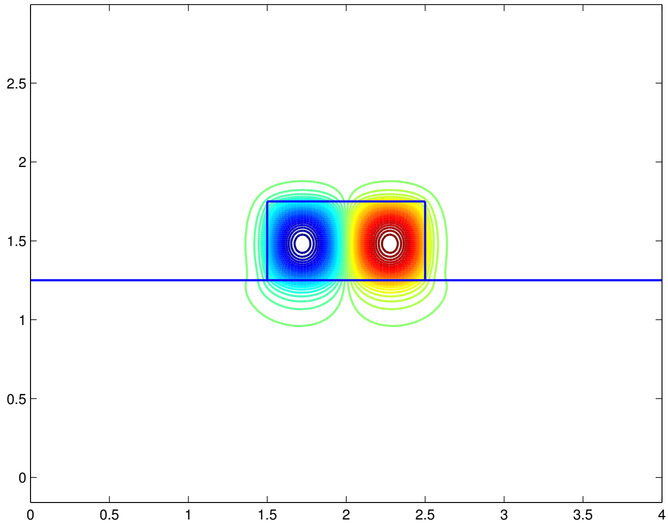
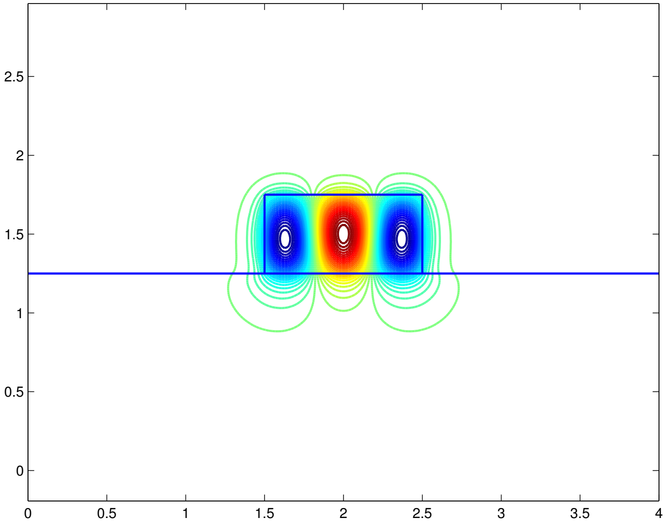
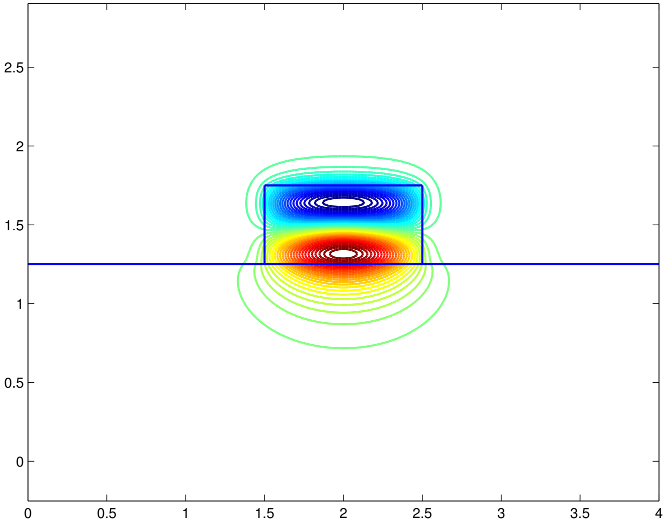


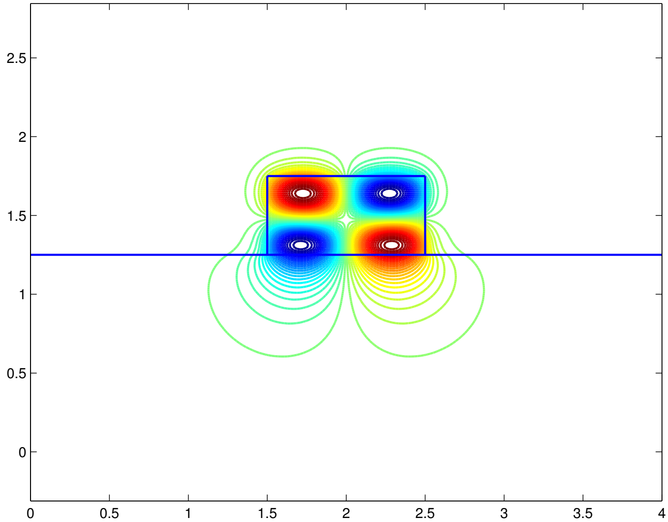
Its first excited mode, TE1



TE2



TE3



TE4