Faraday effect

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Overview

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- optical medium in an external magnetic field

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- Drude model

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- Magneto-optic devices

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- solution is easy with circularly polarized light

Paul Drude, German physicist, 1863-1906

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- right and left handed circular polarization

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- physical dimensions ok.

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- K in $\Delta\chi_{ij} = \mathrm{i}K\epsilon_{ijk}\mathcal{B}_k$ the same as in Drude model!

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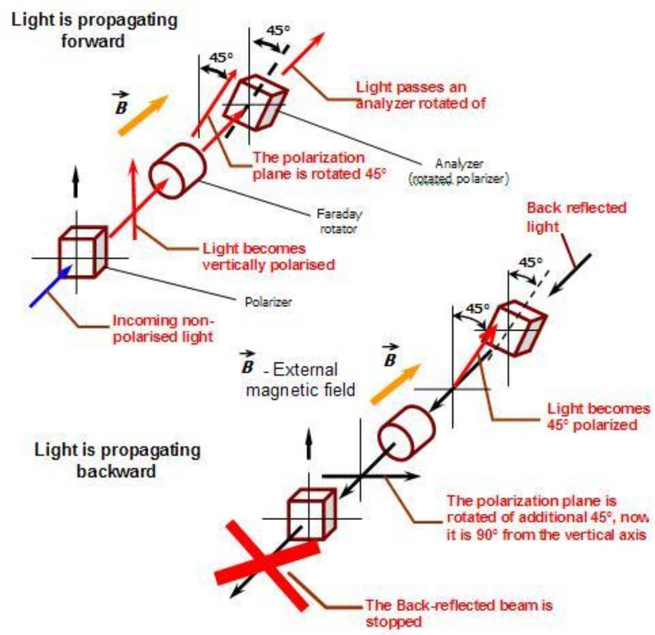
- specific Faraday rotation

$$\Phi_\mathrm{F} = \frac{2\pi}{\lambda}\frac{K\mathcal{B}}{2n}$$

Michael Faraday, English physicist, 1791-1867



polarizer - Faraday rotation by 45 degrees - polarizer chain

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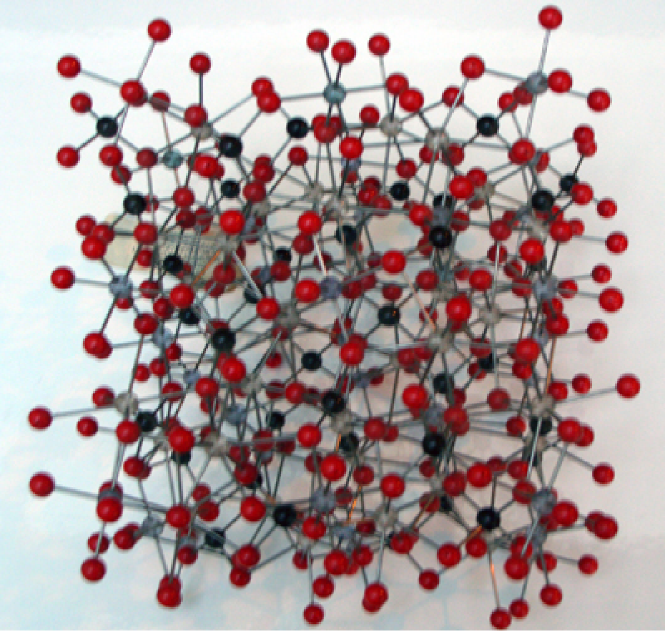
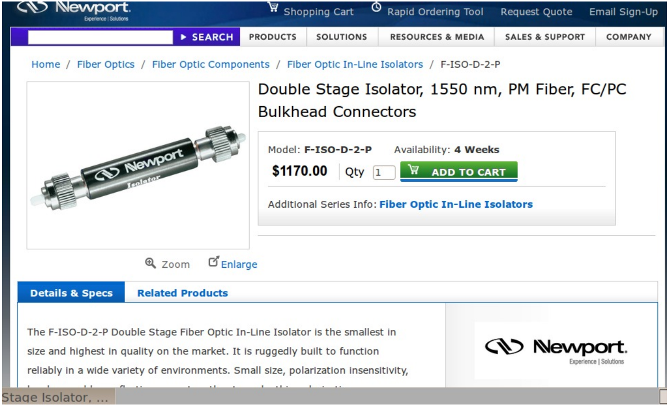
- an ongoing effort



The YIG crystal



A micro-optical isolator. You may buy it.

This one is made in China and costs a few hundred dollars.