Electro- and magnetooptic effects and spatial dispersion

Peter Hertel

University of Osnabrück, Germany

April 13, 2010

Optics

- Neglect absorption $\chi''_{ij}(\omega)$

Optics

- Neglect absorption $\chi''_{ij}(\omega)$
- i.e. $\chi_{ij}(\omega) = \chi^{*}_{ji}(\omega)$

Optics

- Neglect absorption $\chi''_{ij}(\omega)$
- i.e. $\chi_{ij}(\omega) = \chi^*_{ji}(\omega)$
- drop (ω)

Optics

- Neglect absorption $\chi''_{ij}(\omega)$
- i.e. $\chi_{ij}(\omega) = \chi^*_{ji}(\omega)$
- drop (ω)
- $\chi_{ij}(E, B)$

Optics

- Neglect absorption $\chi''_{ij}(\omega)$
- i.e. $\chi_{ij}(\omega) = \chi^*_{ji}(\omega)$
- drop (ω)
- $\chi_{ij}(E, B)$
- quasi-static external fields E_i and B_i

Optics

- Neglect absorption $\chi''_{ij}(\omega)$
- i.e. $\chi_{ij}(\omega) = \chi^*_{ji}(\omega)$
- drop (ω)
- $\chi_{ij}(E, B)$
- quasi-static external fields E_i and B_i
- Onsager: $\chi_{ij}(E, B) = \chi_{ji}(E, -B)$

Optics

- Neglect absorption $\chi''_{ij}(\omega)$
- i.e. $\chi_{ij}(\omega) = \chi^*_{ji}(\omega)$
- drop (ω)
- $\chi_{ij}(E, B)$
- quasi-static external fields E_i and B_i
- Onsager: $\chi_{ij}(E, B) = \chi_{ji}(E, -B)$
- Expand into powers of E and B

Optics

- Neglect absorption $\chi''_{ij}(\omega)$
- i.e. $\chi_{ij}(\omega) = \chi^*_{ji}(\omega)$
- drop (ω)
- $\chi_{ij}(E, B)$
- quasi-static external fields E_i and B_i
- Onsager: $\chi_{ij}(E, B) = \chi_{ji}(E, -B)$
- Expand into powers of E and B
- $\chi = \chi^{0,0} + \chi^{1,0}E + \chi^{0,1}B + \dots$

Optics

- Neglect absorption $\chi''_{ij}(\omega)$
- i.e. $\chi_{ij}(\omega) = \chi^*_{ji}(\omega)$
- drop (ω)
- $\chi_{ij}(E, B)$
- quasi-static external fields E_i and B_i
- Onsager: $\chi_{ij}(E, B) = \chi_{ji}(E, -B)$
- Expand into powers of E and B
- $\chi = \chi^{0,0} + \chi^{1,0} E + \chi^{0,1} B + \dots$
- Einstein's summation convention

Optics

- Neglect absorption $\chi''_{ij}(\omega)$
- i.e. $\chi_{ij}(\omega) = \chi^*_{ji}(\omega)$
- drop (ω)
- $\chi_{ij}(E, B)$
- quasi-static external fields E_i and B_i
- Onsager: $\chi_{ij}(E, B) = \chi_{ji}(E, -B)$
- Expand into powers of E and B
- $\chi = \chi^{0,0} + \chi^{1,0} E + \chi^{0,1} B + \dots$
- Einstein's summation convention
- $\mathbf{a} \cdot \mathbf{b} = \sum_{i=1}^{3} a_i b_i = a_i b_i$

Crystal optics

- Plane wave $\mathbf{F}(t, \mathbf{x}) = \mathbf{f}\, e^{-i\omega t}\, e^{ink_0 \mathbf{w} \cdot \mathbf{x}}$

Crystal optics

- Plane wave $\mathbf{F}(t,\mathbf{x}) = \mathbf{f}\, e^{-i\omega t}\, e^{ink_0\mathbf{w}\cdot\mathbf{x}}$
- Maxwell's equations are

Crystal optics

- Plane wave $\mathbf{F}(t,\mathbf{x}) = \mathbf{f}\, e^{-i\omega t}\, e^{ink_0\mathbf{w}\cdot\mathbf{x}}$
- Maxwell's equations are
- $c\mathbf{b} = -n\mathbf{w} \times (\epsilon\mathbf{e})$ and $\mathbf{e} = n\mathbf{w} \times (c\mathbf{b})$

Crystal optics

- Plane wave $\mathbf{F}(t,\mathbf{x}) = \mathbf{f}\, e^{-i\omega t}\, e^{ink_0\mathbf{w}\cdot\mathbf{x}}$
- Maxwell's equations are
- $c\mathbf{b} = -n\mathbf{w} \times (\epsilon\mathbf{e})$ and $\mathbf{e} = n\mathbf{w} \times (c\mathbf{b})$
- We write W for $\mathbf{w}\times$, $W_{12} = w_3$ etc.

Crystal optics

- Plane wave $\mathbf{F}(t,\mathbf{x}) = \mathbf{f}\, e^{-i\omega t}\, e^{ink_0 \mathbf{w}\cdot\mathbf{x}}$
- Maxwell's equations are
- $c\mathbf{b} = -n\mathbf{w}\times(\epsilon\mathbf{e})$ and $\mathbf{e} = n\mathbf{w}\times(c\mathbf{b})$
- We write W for $\mathbf{w}\times$, $W_{12} = w_3$ etc.
- and eliminate the induction:

Crystal optics

- Plane wave $\mathbf{F}(t,\mathbf{x}) = \mathbf{f}\, e^{-i\omega t}\, e^{ink_0\mathbf{w}\cdot\mathbf{x}}$
- Maxwell's equations are
- $c\mathbf{b} = -n\mathbf{w} \times (\epsilon\mathbf{e})$ and $\mathbf{e} = n\mathbf{w} \times (c\mathbf{b})$
- We write W for $\mathbf{w}\times$, $W_{12} = w_3$ etc.
- and eliminate the induction:
- $\epsilon^{-1}W^2\mathbf{e} = n^{-2}\mathbf{e}$

Crystal optics

- Plane wave $\mathbf{F}(t,\mathbf{x}) = \mathbf{f}\, e^{-i\omega t}\, e^{ink_0\mathbf{w}\cdot\mathbf{x}}$
- Maxwell's equations are
- $c\mathbf{b} = -n\mathbf{w} \times (\epsilon\mathbf{e})$ and $\mathbf{e} = n\mathbf{w} \times (c\mathbf{b})$
- We write W for $\mathbf{w}\times$, $W_{12} = w_3$ etc.
- and eliminate the induction:
- $\epsilon^{-1} W^2 \mathbf{e} = n^{-2}\mathbf{e}$
- Because of $W\mathbf{w} = 0$ there is an unphysical solution $\mathbf{e} \propto \mathbf{w}$

Crystal optics

- Plane wave $\mathbf{F}(t,\mathbf{x}) = \mathbf{f}\, e^{-i\omega t}\, e^{ink_0\mathbf{w}\cdot\mathbf{x}}$
- Maxwell's equations are
- $c\mathbf{b} = -n\mathbf{w} \times (\epsilon\mathbf{e})$ and $\mathbf{e} = n\mathbf{w} \times (c\mathbf{b})$
- We write W for $\mathbf{w}\times$, $W_{12} = w_3$ etc.
- and eliminate the induction:
- $\epsilon^{-1} W^2 \mathbf{e} = n^{-2}\mathbf{e}$
- Because of $W\mathbf{w} = 0$ there is an unphysical solution $\mathbf{e} \propto \mathbf{w}$
- Remaining two eigenvectors describe the polarization of the plane wave

Crystal optics

- Plane wave $\mathbf{F}(t,\mathbf{x}) = \mathbf{f}\, e^{-i\omega t}\, e^{ink_0\mathbf{w}\cdot\mathbf{x}}$
- Maxwell's equations are
- $c\mathbf{b} = -n\mathbf{w}\times(\epsilon\mathbf{e})$ and $\mathbf{e} = n\mathbf{w}\times(c\mathbf{b})$
- We write W for $\mathbf{w}\times$, $W_{12} = w_3$ etc.
- and eliminate the induction:
- $\epsilon^{-1}W^2\mathbf{e} = n^{-2}\mathbf{e}$
- Because of $W\mathbf{w} = 0$ there is an unphysical solution $\mathbf{e} \propto \mathbf{w}$
- Remaining two eigenvectors describe the polarization of the plane wave
- corresponding eigenvalues n^{-2} the refractive indexes

No external fields

- $\chi_{ij}^{0,0}$ is real and symmetric, ϵ_{ij} as well

No external fields

- $\chi_{ij}^{0,0}$ is real and symmetric, ϵ_{ij} as well
- can be diagonalized by an orthogonal transformation

No external fields

- $\chi^{0,0}_{ij}$ is real and symmetric, ϵ_{ij} as well
- can be diagonalized by an orthogonal transformation
- Optically isotropic media

No external fields

- $\chi_{ij}^{0,0}$ is real and symmetric, ϵ_{ij} as well
- can be diagonalized by an orthogonal transformation
- Optically isotropic media
- e.g. glass, cubic crystals such as NaCl

No external fields

- $\chi_{ij}^{0,0}$ is real and symmetric, ϵ_{ij} as well
- can be diagonalized by an orthogonal transformation
- Optically isotropic media
- e.g. glass, cubic crystals such as NaCl
- $\chi_{ij}^{0,0} = \chi\delta_{ij}$

No external fields

- $\chi^{0,0}_{ij}$ is real and symmetric, ϵ_{ij} as well
- can be diagonalized by an orthogonal transformation
- Optically isotropic media
- e.g. glass, cubic crystals such as NaCl
- $\chi^{0,0}_{ij} = \chi\delta_{ij}$
- All transversally polarized plane waves are allowed

No external fields

- $\chi^{0,0}_{ij}$ is real and symmetric, ϵ_{ij} as well
- can be diagonalized by an orthogonal transformation
- Optically isotropic media
- e.g. glass, cubic crystals such as NaCl
- $\chi^{0,0}_{ij} = \chi\delta_{ij}$
- All transversally polarized plane waves are allowed
- Refractive index is $n = \sqrt{\epsilon} = \sqrt{1+\chi}$

No external fields

- $\chi^{0,0}_{ij}$ is real and symmetric, ϵ_{ij} as well
- can be diagonalized by an orthogonal transformation
- Optically isotropic media
- e.g. glass, cubic crystals such as NaCl
- $\chi^{0,0}_{ij} = \chi\delta_{ij}$
- All transversally polarized plane waves are allowed
- Refractive index is $n = \sqrt{\epsilon} = \sqrt{1+\chi}$
- Optically uniaxial crystals:

No external fields

- $\chi_{ij}^{0,0}$ is real and symmetric, ϵ_{ij} as well
- can be diagonalized by an orthogonal transformation
- Optically isotropic media
- e.g. glass, cubic crystals such as NaCl
- $\chi_{ij}^{0,0} = \chi \delta_{ij}$
- All transversally polarized plane waves are allowed
- Refractive index is $n = \sqrt{\epsilon} = \sqrt{1 + \chi}$
- Optically uniaxial crystals:
- two eigenvalues of $\chi_{ij}^{0,0}$ are equal and differ from the third

No external fields

- $\chi_{ij}^{0,0}$ is real and symmetric, ϵ_{ij} as well
- can be diagonalized by an orthogonal transformation
- Optically isotropic media
- e.g. glass, cubic crystals such as NaCl
- $\chi_{ij}^{0,0} = \chi\delta_{ij}$
- All transversally polarized plane waves are allowed
- Refractive index is $n = \sqrt{\epsilon} = \sqrt{1+\chi}$
- Optically uniaxial crystals:
- two eigenvalues of $\chi_{ij}^{0,0}$ are equal and differ from the third
- e.g. lithium niobate

No external fields

- $\chi_{ij}^{0,0}$ is real and symmetric, ϵ_{ij} as well
- can be diagonalized by an orthogonal transformation
- Optically isotropic media
- e.g. glass, cubic crystals such as NaCl
- $\chi_{ij}^{0,0} = \chi\delta_{ij}$
- All transversally polarized plane waves are allowed
- Refractive index is $n = \sqrt{\epsilon} = \sqrt{1+\chi}$
- Optically uniaxial crystals:
- two eigenvalues of $\chi_{ij}^{0,0}$ are equal and differ from the third
- e.g. lithium niobate
- there is a preferred axis and, orthogonal to it, a preferred plane

No external fields (ctd.)

- Optically uniaxial crystals:

No external fields (ctd.)

- Optically uniaxial crystals:
- ordinary refractive index n^0 if polarized in preferred plane

No external fields (ctd.)

- Optically uniaxial crystals:
- ordinary refractive index n^0 if polarized in preferred plane
- $n^0 = \sqrt{\epsilon}$ where ϵ is the doubly occurring eigenvalue

No external fields (ctd.)

- Optically uniaxial crystals:
- ordinary refractive index n^0 if polarized in preferred plane
- $n^0 = \sqrt{\epsilon}$ where ϵ is the doubly occurring eigenvalue
- extraordinary refractive index n^e if polarized in preferred axis

No external fields (ctd.)

- Optically uniaxial crystals:
- ordinary refractive index n^0 if polarized in preferred plane
- $n^0 = \sqrt{\epsilon}$ where ϵ is the doubly occurring eigenvalue
- extraordinary refractive index n^e if polarized in preferred axis
- If light travels along the preferred (optical) axis, it can be arbitrarily polarized

No external fields (ctd.)

- Optically uniaxial crystals:
- ordinary refractive index n^0 if polarized in preferred plane
- $n^0 = \sqrt{\epsilon}$ where ϵ is the doubly occurring eigenvalue
- extraordinary refractive index n^e if polarized in preferred axis
- If light travels along the preferred (optical) axis, it can be arbitrarily polarized
- If a beam enters crystal at an arbitrary angle, it splits into an ordinary and an extraordinary beam.

No external fields (ctd.)

- Optically uniaxial crystals:
- ordinary refractive index n^0 if polarized in preferred plane
- $n^0 = \sqrt{\epsilon}$ where ϵ is the doubly occurring eigenvalue
- extraordinary refractive index n^e if polarized in preferred axis
- If light travels along the preferred (optical) axis, it can be arbitrarily polarized
- If a beam enters crystal at an arbitrary angle, it splits into an ordinary and an extraordinary beam.
- Birefringence

No external fields (ctd.)

- Optically uniaxial crystals:
- ordinary refractive index n^0 if polarized in preferred plane
- $n^0 = \sqrt{\epsilon}$ where ϵ is the doubly occurring eigenvalue
- extraordinary refractive index n^e if polarized in preferred axis
- If light travels along the preferred (optical) axis, it can be arbitrarily polarized
- If a beam enters crystal at an arbitrary angle, it splits into an ordinary and an extraordinary beam.
- Birefringence
- Optically biaxial crystal:

No external fields (ctd.)

- Optically uniaxial crystals:
- ordinary refractive index n^0 if polarized in preferred plane
- $n^0 = \sqrt{\epsilon}$ where ϵ is the doubly occurring eigenvalue
- extraordinary refractive index n^e if polarized in preferred axis
- If light travels along the preferred (optical) axis, it can be arbitrarily polarized
- If a beam enters crystal at an arbitrary angle, it splits into an ordinary and an extraordinary beam.
- Birefringence
- Optically biaxial crystal:
- all three eigenvalues of ϵ_{ij} are different

No external fields (ctd.)

- Optically uniaxial crystals:
- ordinary refractive index n^0 if polarized in preferred plane
- $n^0 = \sqrt{\epsilon}$ where ϵ is the doubly occurring eigenvalue
- extraordinary refractive index n^e if polarized in preferred axis
- If light travels along the preferred (optical) axis, it can be arbitrarily polarized
- If a beam enters crystal at an arbitrary angle, it splits into an ordinary and an extraordinary beam.
- Birefringence
- Optically biaxial crystal:
- all three eigenvalues of ϵ_{ij} are different
- e.g. potassium niobate

No external fields (ctd.)

- Optically uniaxial crystals:
- ordinary refractive index n^0 if polarized in preferred plane
- $n^0 = \sqrt{\epsilon}$ where ϵ is the doubly occurring eigenvalue
- extraordinary refractive index n^e if polarized in preferred axis
- If light travels along the preferred (optical) axis, it can be arbitrarily polarized
- If a beam enters crystal at an arbitrary angle, it splits into an ordinary and an extraordinary beam.
- Birefringence
- Optically biaxial crystal:
- all three eigenvalues of ϵ_{ij} are different
- e.g. potassium niobate
- two optical axes

Pockels effect

- contribution $\delta\chi_{ij} = \chi^{1,0}_{ijk} E_k$

Pockels effect

- contribution $\delta\chi_{ij} = \chi^{1,0}_{ijk} E_k$
- $\Delta\chi_{ij}$ must be be hermitian and symmetric, hence real

Pockels effect

- contribution $\delta\chi_{ij} = \chi^{1,0}_{ijk} E_k$
- $\Delta\chi_{ij}$ must be be hermitian and symmetric, hence real
- Maxwell's equations are invariant with respect to space inversion

Pockels effect

- contribution $\delta\chi_{ij} = \chi^{1,0}_{ijk} E_k$
- $\Delta\chi_{ij}$ must be be hermitian and symmetric, hence real
- Maxwell's equations are invariant with respect to space inversion
- $t, \mathbf{x}, \rho, \mathbf{j}, \mathbf{E}, \mathbf{B}$ goes to $t, -\mathbf{x}, \rho, -\mathbf{j}, -\mathbf{E}, \mathbf{B}$

Pockels effect

- contribution $\delta\chi_{ij} = \chi^{1,0}_{ijk} E_k$
- $\Delta\chi_{ij}$ must be be hermitian and symmetric, hence real
- Maxwell's equations are invariant with respect to space inversion
- $t, \mathbf{x}, \rho, \mathbf{j}, \mathbf{E}, \mathbf{B}$ goes to $t, -\mathbf{x}, \rho, -\mathbf{j}, -\mathbf{E}, \mathbf{B}$
- $\epsilon_0 \boldsymbol{\nabla} \cdot \mathbf{E} = \rho$ and $(1/\mu_0)\boldsymbol{\nabla} \times \mathbf{B} - \epsilon_0 \dot{\mathbf{E}} = \mathbf{j}$

Pockels effect

- contribution $\delta\chi_{ij} = \chi^{1,0}_{ijk} E_k$
- $\Delta\chi_{ij}$ must be be hermitian and symmetric, hence real
- Maxwell's equations are invariant with respect to space inversion
- $t, \mathbf{x}, \rho, \mathbf{j}, \mathbf{E}, \mathbf{B}$ goes to $t, -\mathbf{x}, \rho, -\mathbf{j}, -\mathbf{E}, \mathbf{B}$
- $\epsilon_0 \boldsymbol{\nabla} \cdot \mathbf{E} = \rho$ and $(1/\mu_0)\boldsymbol{\nabla} \times \mathbf{B} - \epsilon_0 \dot{\mathbf{E}} = \mathbf{j}$
- $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ and $\boldsymbol{\nabla} \times \mathbf{E} + \dot{\mathbf{B}} = 0$

Pockels effect

- contribution $\delta\chi_{ij} = \chi^{1,0}_{ijk} E_k$
- $\Delta\chi_{ij}$ must be be hermitian and symmetric, hence real
- Maxwell's equations are invariant with respect to space inversion
- $t, \mathbf{x}, \rho, \mathbf{j}, \mathbf{E}, \mathbf{B}$ goes to $t, -\mathbf{x}, \rho, -\mathbf{j}, -\mathbf{E}, \mathbf{B}$
- $\epsilon_0 \boldsymbol{\nabla} \cdot \mathbf{E} = \rho$ and $(1/\mu_0)\boldsymbol{\nabla} \times \mathbf{B} - \epsilon_0 \dot{\mathbf{E}} = \mathbf{j}$
- $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ and $\boldsymbol{\nabla} \times \mathbf{E} + \dot{\mathbf{B}} = 0$
- If there is an space inversion center, $\chi^{1,0}_{ijk}$ must coincide with negative, hence vanish.

Pockels effect

- contribution $\delta\chi_{ij} = \chi^{1,0}_{ijk} E_k$
- $\Delta\chi_{ij}$ must be be hermitian and symmetric, hence real
- Maxwell's equations are invariant with respect to space inversion
- $t, \mathbf{x}, \rho, \mathbf{j}, \mathbf{E}, \mathbf{B}$ goes to $t, -\mathbf{x}, \rho, -\mathbf{j}, -\mathbf{E}, \mathbf{B}$
- $\epsilon_0 \boldsymbol{\nabla} \cdot \mathbf{E} = \rho$ and $(1/\mu_0)\boldsymbol{\nabla} \times \mathbf{B} - \epsilon_0 \dot{\mathbf{E}} = \mathbf{j}$
- $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ and $\boldsymbol{\nabla} \times \mathbf{E} + \dot{\mathbf{B}} = 0$
- If there is an space inversion center, $\chi^{1,0}_{ijk}$ must coincide with negative, hence vanish.
- Pockels effect only for crystals without inversion center

Pockels effect

- contribution $\delta\chi_{ij} = \chi^{1,0}_{ijk} E_k$
- $\Delta\chi_{ij}$ must be be hermitian and symmetric, hence real
- Maxwell's equations are invariant with respect to space inversion
- $t, \mathbf{x}, \rho, \mathbf{j}, \mathbf{E}, \mathbf{B}$ goes to $t, -\mathbf{x}, \rho, -\mathbf{j}, -\mathbf{E}, \mathbf{B}$
- $\epsilon_0 \boldsymbol{\nabla} \cdot \mathbf{E} = \rho$ and $(1/\mu_0)\boldsymbol{\nabla} \times \mathbf{B} - \epsilon_0 \dot{\mathbf{E}} = \mathbf{j}$
- $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ and $\boldsymbol{\nabla} \times \mathbf{E} + \dot{\mathbf{B}} = 0$
- If there is an space inversion center, $\chi^{1,0}_{ijk}$ must coincide with negative, hence vanish.
- Pockels effect only for crystals without inversion center
- e.g. lithium niobate

Pockels effect

- contribution $\delta\chi_{ij} = \chi^{1,0}_{ijk} E_k$
- $\Delta\chi_{ij}$ must be be hermitian and symmetric, hence real
- Maxwell's equations are invariant with respect to space inversion
- $t, \mathbf{x}, \rho, \mathbf{j}, \mathbf{E}, \mathbf{B}$ goes to $t, -\mathbf{x}, \rho, -\mathbf{j}, -\mathbf{E}, \mathbf{B}$
- $\epsilon_0 \boldsymbol{\nabla} \cdot \mathbf{E} = \rho$ and $(1/\mu_0)\boldsymbol{\nabla} \times \mathbf{B} - \epsilon_0 \dot{\mathbf{E}} = \mathbf{j}$
- $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ and $\boldsymbol{\nabla} \times \mathbf{E} + \dot{\mathbf{B}} = 0$
- If there is an space inversion center, $\chi^{1,0}_{ijk}$ must coincide with negative, hence vanish.
- Pockels effect only for crystals without inversion center
- e.g. lithium niobate
- Pockels effect allows to efficiently modulate and switch light

Friedrich Carl Alwin Pockels

Faraday effect

- contribution $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k$

Faraday effect

- contribution $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k$
- $\Delta\chi_{ij}$ must be hermitian and antisymmetric, hence purely imaginary

Faraday effect

- contribution $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k$
- $\Delta\chi_{ij}$ must be hermitian and antisymmetric, hence purely imaginary
- e.g. yttrium iron garnet (YIG), a ferri-magnet

Faraday effect

- contribution $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k$
- $\Delta\chi_{ij}$ must be hermitian and antisymmetric, hence purely imaginary
- e.g. yttrium iron garnet (YIG), a ferri-magnet
- $\mathbf{B} = \mu_0 \mathbf{M}$

Faraday effect

- contribution $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k$
- $\Delta\chi_{ij}$ must be hermitian and antisymmetric, hence purely imaginary
- e.g. yttrium iron garnet (YIG), a ferri-magnet
- $\mathbf{B} = \mu_0 \mathbf{M}$
- $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k = iK\epsilon_{ijk} M_k$

Faraday effect

- contribution $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k$
- $\Delta\chi_{ij}$ must be hermitian and antisymmetric, hence purely imaginary
- e.g. yttrium iron garnet (YIG), a ferri-magnet
- $\mathbf{B} = \mu_0 \mathbf{M}$
- $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k = iK\epsilon_{ijk} M_k$
- Because of Faraday effect, left handed and right handed circularly polarized light beams propagate with different refractive indexes n_L and n_R

Faraday effect

- contribution $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k$
- $\Delta\chi_{ij}$ must be hermitian and antisymmetric, hence purely imaginary
- e.g. yttrium iron garnet (YIG), a ferri-magnet
- $\mathbf{B} = \mu_0 \mathbf{M}$
- $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k = iK\epsilon_{ijk} M_k$
- Because of Faraday effect, left handed and right handed circularly polarized light beams propagate with different refractive indexes n_L and n_R
- When incident on the crystal, a linearly polarized wave splits into a superposition of left handed and right handed circularly polarized light

Faraday effect

- contribution $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k$
- $\Delta\chi_{ij}$ must be hermitian and antisymmetric, hence purely imaginary
- e.g. yttrium iron garnet (YIG), a ferri-magnet
- $\mathbf{B} = \mu_0 \mathbf{M}$
- $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k = iK\epsilon_{ijk} M_k$
- Because of Faraday effect, left handed and right handed circularly polarized light beams propagate with different refractive indexes n_L and n_R
- When incident on the crystal, a linearly polarized wave splits into a superposition of left handed and right handed circularly polarized light
- When exiting the crystal, they superimpose to a linearly polarized beam

Faraday effect

- contribution $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k$
- $\Delta\chi_{ij}$ must be hermitian and antisymmetric, hence purely imaginary
- e.g. yttrium iron garnet (YIG), a ferri-magnet
- $\mathbf{B} = \mu_0 \mathbf{M}$
- $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k = iK\epsilon_{ijk} M_k$
- Because of Faraday effect, left handed and right handed circularly polarized light beams propagate with different refractive indexes n_L and n_R
- When incident on the crystal, a linearly polarized wave splits into a superposition of left handed and right handed circularly polarized light
- When exiting the crystal, they superimpose to a linearly polarized beam
- However, polarization has been rotated

Faraday effect

- contribution $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k$
- $\Delta\chi_{ij}$ must be hermitian and antisymmetric, hence purely imaginary
- e.g. yttrium iron garnet (YIG), a ferri-magnet
- $\mathbf{B} = \mu_0 \mathbf{M}$
- $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k = iK\epsilon_{ijk} M_k$
- Because of Faraday effect, left handed and right handed circularly polarized light beams propagate with different refractive indexes n_L and n_R
- When incident on the crystal, a linearly polarized wave splits into a superposition of left handed and right handed circularly polarized light
- When exiting the crystal, they superimpose to a linearly polarized beam
- However, polarization has been rotated
- Faraday effect is non-reciprocal

Faraday effect

- contribution $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k$
- $\Delta\chi_{ij}$ must be hermitian and antisymmetric, hence purely imaginary
- e.g. yttrium iron garnet (YIG), a ferri-magnet
- $\mathbf{B} = \mu_0 \mathbf{M}$
- $\Delta\chi_{ij} = \chi^{0,1}_{ijk} B_k = iK\epsilon_{ijk} M_k$
- Because of Faraday effect, left handed and right handed circularly polarized light beams propagate with different refractive indexes n_L and n_R
- When incident on the crystal, a linearly polarized wave splits into a superposition of left handed and right handed circularly polarized light
- When exiting the crystal, they superimpose to a linearly polarized beam
- However, polarization has been rotated
- Faraday effect is non-reciprocal

Faraday effect (ctd.)

- Assume polarized beam in the direction of magnetization, say 3-axis, $k_0 = \omega/c$, $\mathbf{k} = nk_0(0, 0, 1)$

Faraday effect (ctd.)

- Assume polarized beam in the direction of magnetization, say 3-axis, $k_0 = \omega/c$, $\mathbf{k} = nk_0(0,0,1)$
- solve $-\mathbf{k} \times \mathbf{k} \times \mathbf{E} = k_0^2\,\epsilon\,\mathbf{E}$

Faraday effect (ctd.)

- Assume polarized beam in the direction of magnetization, say 3-axis, $k_0 = \omega/c$, $\mathbf{k} = nk_0(0,0,1)$
- solve $-\mathbf{k} \times \mathbf{k} \times \mathbf{E} = k_0^2\, \epsilon\, \mathbf{E}$
- pemittivity tensor is

$$\epsilon = \begin{pmatrix} \epsilon & iKM & 0 \\ -iKM & \epsilon & 0 \\ 0 & 0 & \epsilon \end{pmatrix}$$

Faraday effect (ctd.)

- Assume polarized beam in the direction of magnetization, say 3-axis, $k_0 = \omega/c$, $\mathbf{k} = nk_0(0,0,1)$
- solve $-\mathbf{k} \times \mathbf{k} \times \mathbf{E} = k_0^2\, \epsilon\, \mathbf{E}$
- pemittivity tensor is

$$\epsilon = \begin{pmatrix} \epsilon & iKM & 0 \\ -iKM & \epsilon & 0 \\ 0 & 0 & \epsilon \end{pmatrix}$$

- solutions are

$$\mathbf{e}_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} \text{ and } \mathbf{e}_R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$

Faraday effect (ctd.)

- Assume polarized beam in the direction of magnetization, say 3-axis, $k_0 = \omega/c$, $\mathbf{k} = nk_0(0, 0, 1)$
- solve $-\mathbf{k} \times \mathbf{k} \times \mathbf{E} = k_0^2\, \epsilon\, \mathbf{E}$
- pemittivity tensor is

$$\epsilon = \begin{pmatrix} \epsilon & iKM & 0 \\ -iKM & \epsilon & 0 \\ 0 & 0 & \epsilon \end{pmatrix}$$

- solutions are

$$\mathbf{e}_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} \text{ and } \mathbf{e}_R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$

- refractive indexes $n_L^2 = \epsilon + KM$ and $n_R^2 = \epsilon - KM$

Faraday effect (ctd.)

- Assume polarized beam in the direction of magnetization, say 3-axis, $k_0 = \omega / c$, $\mathbf{k} = n k_0 (0, 0, 1)$
- solve $-\mathbf{k} \times \mathbf{k} \times \mathbf{E} = k_0^2 \, \epsilon \, \mathbf{E}$
- pemittivity tensor is

$$\epsilon = \begin{pmatrix} \epsilon & iKM & 0 \\ -iKM & \epsilon & 0 \\ 0 & 0 & \epsilon \end{pmatrix}$$

- solutions are

$$\mathbf{e}_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} \text{ and } \mathbf{e}_R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix}$$

- refractive indexes $n_L^2 = \epsilon + KM$ and $n_R^2 = \epsilon - KM$
- specific Faraday rotation constant is

$$\Theta = \frac{2\pi}{\ } \frac{KM}{\ }$$

Michael Faraday

Kerr effect

- contribution $\Delta\chi_{ij} = \chi^{2,0}_{ijkl} E_k E_l$

Kerr effect

- contribution $\Delta\chi_{ij} = \chi^{2,0}_{ijkl} E_k E_l$
- much weaker than Pockels effect, if the latter is allowed

Kerr effect

- contribution $\Delta\chi_{ij} = \chi^{2,0}_{ijkl} E_k E_l$
- much weaker than Pockels effect, if the latter is allowed
- possible even for isotropic media

Kerr effect

- contribution $\Delta\chi_{ij} = \chi^{2,0}_{ijkl} E_k E_l$
- much weaker than Pockels effect, if the latter is allowed
- possible even for isotropic media
- $\chi^{2,0}_{ijkl} = \gamma \delta_{ij} \delta_{kl}$

Kerr effect

- contribution $\Delta\chi_{ij} = \chi^{2,0}_{ijkl} E_k E_l$
- much weaker than Pockels effect, if the latter is allowed
- possible even for isotropic media
- $\chi^{2,0}_{ijkl} = \gamma \delta_{ij} \delta_{kl}$
- i.e. $\Delta\chi_{ij} = \gamma \delta_{ij} |\mathbf{E}|^2$

Kerr effect

- contribution $\Delta\chi_{ij} = \chi^{2,0}_{ijkl} E_k E_l$
- much weaker than Pockels effect, if the latter is allowed
- possible even for isotropic media
- $\chi^{2,0}_{ijkl} = \gamma\delta_{ij}\delta_{kl}$
- i.e. $\Delta\chi_{ij} = \gamma\delta_{ij}|\mathbf{E}|^2$
- Note, however, that this formula is also applied if E is a rapidly oscillating field

Kerr effect

- contribution $\Delta\chi_{ij} = \chi^{2,0}_{ijkl} E_k E_l$
- much weaker than Pockels effect, if the latter is allowed
- possible even for isotropic media
- $\chi^{2,0}_{ijkl} = \gamma\delta_{ij}\delta_{kl}$
- i.e. $\Delta\chi_{ij} = \gamma\delta_{ij}|\mathbf{E}|^2$
- Note, however, that this formula is also applied if E is a rapidly oscillating field
- Should be studied by Nonlinear Response Theory

Kerr effect

- contribution $\Delta\chi_{ij} = \chi^{2,0}_{ijkl} E_k E_l$
- much weaker than Pockels effect, if the latter is allowed
- possible even for isotropic media
- $\chi^{2,0}_{ijkl} = \gamma \delta_{ij} \delta_{kl}$
- i.e. $\Delta\chi_{ij} = \gamma \delta_{ij} |\mathbf{E}|^2$
- Note, however, that this formula is also applied if E is a rapidly oscillating field
- Should be studied by Nonlinear Response Theory
- also $\chi^{2,0}_{ijkl} \propto \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}$

Kerr effect

- contribution $\Delta\chi_{ij} = \chi^{2,0}_{ijkl} E_k E_l$
- much weaker than Pockels effect, if the latter is allowed
- possible even for isotropic media
- $\chi^{2,0}_{ijkl} = \gamma \delta_{ij} \delta_{kl}$
- i.e. $\Delta\chi_{ij} = \gamma \delta_{ij} |\mathbf{E}|^2$
- Note, however, that this formula is also applied if E is a rapidly oscillating field
- Should be studied by Nonlinear Response Theory
- also $\chi^{2,0}_{ijkl} \propto \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}$
- gives $\Delta\chi_{ij} \propto E_i E_j$, causes birefringence

Kerr effect

- contribution $\Delta\chi_{ij} = \chi^{2,0}_{ijkl} E_k E_l$
- much weaker than Pockels effect, if the latter is allowed
- possible even for isotropic media
- $\chi^{2,0}_{ijkl} = \gamma\delta_{ij}\delta_{kl}$
- i.e. $\Delta\chi_{ij} = \gamma\delta_{ij}|\mathbf{E}|^2$
- Note, however, that this formula is also applied if E is a rapidly oscillating field
- Should be studied by Nonlinear Response Theory
- also $\chi^{2,0}_{ijkl} \propto \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}$
- gives $\Delta\chi_{ij} \propto E_i E_j$, causes birefringence
- Kerr cell

John Kerr

Magneto-electric effect

- contribution $\Delta\chi_{ij} = \chi^{1,1}_{ijkl} E_k B_l$

Magneto-electric effect

- contribution $\Delta\chi_{ij} = \chi^{1,1}_{ijkl} E_k B_l$
- no special name

Magneto-electric effect

- contribution $\Delta\chi_{ij} = \chi^{1,1}_{ijkl} E_k B_l$
- no special name
- Pockels effect depens linearly on magnetic induction

Magneto-electric effect

- contribution $\Delta\chi_{ij} = \chi^{1,1}_{ijkl} E_k B_l$
- no special name
- Pockels effect depens linearly on magnetic induction
- Faraday effect depends linearly on electric field strength

Magneto-electric effect

- contribution $\Delta\chi_{ij} = \chi^{1,1}_{ijkl} E_k B_l$
- no special name
- Pockels effect depens linearly on magnetic induction
- Faraday effect depends linearly on electric field strength
- The crystal must be ferri- or ferromagnetic, transparent, and must not have an inversion center

Magneto-electric effect

- contribution $\Delta\chi_{ij} = \chi^{1,1}_{ijkl} E_k B_l$
- no special name
- Pockels effect depens linearly on magnetic induction
- Faraday effect depends linearly on electric field strength
- The crystal must be ferri- or ferromagnetic, transparent, and must not have an inversion center
- ???

Cotton-Mouton effect

- contribution $\Delta\chi_{ij} = \chi^{0,2}_{ijkl} B_k B_l$

Cotton-Mouton effect

- contribution $\Delta\chi_{ij} = \chi^{0,2}_{ijkl} B_k B_l$
- very weak for paramagnetic substances

Cotton-Mouton effect

- contribution $\Delta\chi_{ij} = \chi^{0,2}_{ijkl} B_k B_l$
- very weak for paramagnetic substances
- but might be strong for ferri- or ferromagnetic crystals

Cotton-Mouton effect

- contribution $\Delta\chi_{ij} = \chi^{0,2}_{ijkl} B_k B_l$
- very weak for paramagnetic substances
- but might be strong for ferri- or ferromagnetic crystals
- $\Delta\chi_{ij} \propto \delta_{ij} |\mathbf{M}|^2$ will escape detection

Cotton-Mouton effect

- contribution $\Delta\chi_{ij} = \chi^{0,2}_{ijkl} B_k B_l$
- very weak for paramagnetic substances
- but might be strong for ferri- or ferromagnetic crystals
- $\Delta\chi_{ij} \propto \delta_{ij} |\mathbf{M}|^2$ will escape detection
- but $\Delta\chi_{ij} \propto M_i M_j$ causes birefringence

Cotton-Mouton effect

- contribution $\Delta\chi_{ij} = \chi^{0,2}_{ijkl} B_k B_l$
- very weak for paramagnetic substances
- but might be strong for ferri- or ferromagnetic crystals
- $\Delta\chi_{ij} \propto \delta_{ij} |\mathbf{M}|^2$ will escape detection
- but $\Delta\chi_{ij} \propto M_i M_j$ causes birefringence
- Cotton-Mouton effect is reciprocal

Cotton-Mouton effect

- contribution $\Delta\chi_{ij} = \chi^{0,2}_{ijkl} B_k B_l$
- very weak for paramagnetic substances
- but might be strong for ferri- or ferromagnetic crystals
- $\Delta\chi_{ij} \propto \delta_{ij} |\mathbf{M}|^2$ will escape detection
- but $\Delta\chi_{ij} \propto M_i M_j$ causes birefringence
- Cotton-Mouton effect is reciprocal
- it does not differentiate between forward and backward propagation

Spatial dispersion

- Recall

$$\chi_{ij}(\omega,\mathbf{q}) = \frac{1}{\epsilon_0}\int_0^\infty d\tau\, e^{\,i\omega\tau}\int d^3\boldsymbol{\xi}\, e^{\,-i\mathbf{q}\cdot\boldsymbol{\xi}}\,\Gamma_{ij}(\tau,\mathbf{q})$$

Spatial dispersion

- Recall

$$\chi_{ij}(\omega,\mathbf{q}) = \frac{1}{\epsilon_0}\int_0^\infty d\tau\, e^{\,i\omega\tau} \int d^3\boldsymbol{\xi}\, e^{\,-i\mathbf{q}\cdot\boldsymbol{\xi}}\, \Gamma_{ij}(\tau,\mathbf{q})$$

- where

$$\Gamma_{ij}(\tau,\boldsymbol{\xi}) = \operatorname{tr} G\,\frac{i}{\hbar}[P_i(\tau,\boldsymbol{\xi}),P_j(0,0)]$$

Spatial dispersion

- Recall

$$\chi_{ij}(\omega,\mathbf{q}) = \frac{1}{\epsilon_0}\int_0^\infty d\tau\, e^{\,i\omega\tau} \int d^3\xi\, e^{\,-i\mathbf{q}\cdot\boldsymbol{\xi}}\,\Gamma_{ij}(\tau,\mathbf{q})$$

- where

$$\Gamma_{ij}(\tau,\boldsymbol{\xi}) = \operatorname{tr} G\,\frac{i}{\hbar}[P_i(\tau,\boldsymbol{\xi}),P_j(0,0)]$$

- $\hat{P}_i(\omega,\mathbf{q}) = \chi_{ij}(\omega,\mathbf{q})\hat{E}_j(\omega,\mathbf{q})$

Spatial dispersion

- Recall

$$\chi_{ij}(\omega,\mathbf{q}) = \frac{1}{\epsilon_0}\int_0^\infty d\tau\, e^{\,i\omega\tau} \int d^3\boldsymbol{\xi}\, e^{\,-i\mathbf{q}\cdot\boldsymbol{\xi}}\, \Gamma_{ij}(\tau,\mathbf{q})$$

- where

$$\Gamma_{ij}(\tau,\boldsymbol{\xi}) = \operatorname{tr} G\,\frac{i}{\hbar}[P_i(\tau,\boldsymbol{\xi}),P_j(0,0)]$$

- $\hat{P}_i(\omega,\mathbf{q}) = \chi_{ij}(\omega,\mathbf{q})\hat{E}_j(\omega,\mathbf{q})$
- In linear approximation, response $\hat{P}_i$ has same angular frequency ω and wave vector $\mathbf{q}$ as perturbation $\hat{E}_j$

Dispersion relation

- The arguments of $\hat{E}_i(\omega, \mathbf{q})$ are not independent

Dispersion relation

- The arguments of $\hat{E}_i(\omega, \mathbf{q})$ are not independent
- after all, there are Maxwell's equations

Dispersion relation

- The arguments of $\hat{E}_i(\omega, \mathbf{q})$ are <u>not</u> independent
- after all, there are Maxwell's equations
- $(\mathbf{q}^2\delta_{ij} - q_i q_j)\hat{E}_j = (\omega/c)^2\,(\delta_{ij} + \chi_{ij})\hat{E}_j$

Dispersion relation

- The arguments of $\hat{E}_i(\omega, \mathbf{q})$ are <u>not</u> independent
- after all, there are Maxwell's equations
- $(\mathbf{q}^2\delta_{ij} - q_i q_j)\hat{E}_j = (\omega/c)^2\,(\delta_{ij} + \chi_{ij})\hat{E}_j$
- both $\hat{E}_j$ <u>and</u> χ_{ij} depend on ω and $\mathbf{q}$

Dispersion relation

- The arguments of $\hat{E}_i(\omega, \mathbf{q})$ are <u>not</u> independent
- after all, there are Maxwell's equations
- $(\mathbf{q}^2\delta_{ij} - q_i q_j)\hat{E}_j = (\omega/c)^2\,(\delta_{ij} + \chi_{ij})\hat{E}_j$
- both $\hat{E}_j$ <u>and</u> χ_{ij} depend on ω and $\mathbf{q}$
- $\mathbf{q}$ given, find ω such that Maxwell's equations are satisfied for a non-trivial eigenvector (polarization)

Dispersion relation

- The arguments of $\hat{E}_i(\omega, \mathbf{q})$ are <u>not</u> independent
- after all, there are Maxwell's equations
- $(\mathbf{q}^2\delta_{ij} - q_i q_j)\hat{E}_j = (\omega/c)^2\,(\delta_{ij} + \chi_{ij})\hat{E}_j$
- both $\hat{E}_j$ <u>and</u> χ_{ij} depend on ω and $\mathbf{q}$
- $\mathbf{q}$ given, find ω such that Maxwell's equations are satisfied for a non-trivial eigenvector (polarization)
- $\omega = \omega(\mathbf{q})$ is a dispersion relation.

Dispersion relation

- The arguments of $\hat{E}_i(\omega, \mathbf{q})$ are not independent
- after all, there are Maxwell's equations
- $(\mathbf{q}^2\delta_{ij} - q_i q_j)\hat{E}_j = (\omega/c)^2\,(\delta_{ij} + \chi_{ij})\hat{E}_j$
- both $\hat{E}_j$ and χ_{ij} depend on ω and $\mathbf{q}$
- $\mathbf{q}$ given, find ω such that Maxwell's equations are satisfied for a non-trivial eigenvector (polarization)
- $\omega = \omega(\mathbf{q})$ is a dispersion relation.
- eigenvalue problem is non-linear, since ω enters multiply

Dispersion relation

- The arguments of $\hat{E}_i(\omega, \mathbf{q})$ are <u>not</u> independent
- after all, there are Maxwell's equations
- $(\mathbf{q}^2\delta_{ij} - q_i q_j)\hat{E}_j = (\omega/c)^2\,(\delta_{ij} + \chi_{ij})\hat{E}_j$
- both $\hat{E}_j$ <u>and</u> χ_{ij} depend on ω and $\mathbf{q}$
- $\mathbf{q}$ given, find ω such that Maxwell's equations are satisfied for a non-trivial eigenvector (polarization)
- $\omega = \omega(\mathbf{q})$ is a dispersion relation.
- eigenvalue problem is non-linear, since ω enters multiply
- if $\chi_{ij}(\omega) = \chi_{ij}(\omega, 0)$ is used, then ω may be considered an indipenden variable and the refractive index n in $\mathbf{q} = nk_0\mathbf{w}$ can be determined

Dispersion relation

- The arguments of $\hat{E}_i(\omega, \mathbf{q})$ are <u>not</u> independent
- after all, there are Maxwell's equations
- $(\mathbf{q}^2\delta_{ij} - q_i q_j)\hat{E}_j = (\omega/c)^2\,(\delta_{ij} + \chi_{ij})\hat{E}_j$
- both $\hat{E}_j$ <u>and</u> χ_{ij} depend on ω and $\mathbf{q}$
- $\mathbf{q}$ given, find ω such that Maxwell's equations are satisfied for a non-trivial eigenvector (polarization)
- $\omega = \omega(\mathbf{q})$ is a dispersion relation.
- eigenvalue problem is non-linear, since ω enters multiply
- if $\chi_{ij}(\omega) = \chi_{ij}(\omega, 0)$ is used, then ω may be considered an indipenden variable and the refractive index n in $\mathbf{q} = nk_0\mathbf{w}$ can be determined
- ...as discussed earlier

Optical activity

- $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ij}(\omega, 0) + \chi^{oa}_{ijk}(\omega) q_k$

Optical activity

- $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ij}(\omega, 0) + \chi^{oa}_{ijk}(\omega) q_k$
- Invariance with respect to time reversal: $\Gamma_{ij}(\tau, \boldsymbol{\xi}) = \Gamma_{ji}(\tau, -\boldsymbol{\xi})$

Optical activity

- $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ij}(\omega, 0) + \chi^{oa}_{ijk}(\omega) q_k$
- Invariance with respect to time reversal: $\Gamma_{ij}(\tau, \boldsymbol{\xi}) = \Gamma_{ji}(\tau, -\boldsymbol{\xi})$
- which implies $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ji}(\omega, -\mathbf{q})$

Optical activity

- $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ij}(\omega, 0) + \chi^{oa}_{ijk}(\omega) q_k$
- Invariance with respect to time reversal: $\Gamma_{ij}(\tau, \boldsymbol{\xi}) = \Gamma_{ji}(\tau, -\boldsymbol{\xi})$
- which implies $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ji}(\omega, -\mathbf{q})$
- if present, a magnetic field must be inverted as well

Optical activity

- $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ij}(\omega, 0) + \chi^{oa}_{ijk}(\omega) q_k$
- Invariance with respect to time reversal: $\Gamma_{ij}(\tau, \boldsymbol{\xi}) = \Gamma_{ji}(\tau, -\boldsymbol{\xi})$
- which implies $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ji}(\omega, -\mathbf{q})$
- if present, a magnetic field must be inverted as well
- recall that χ_{ij} is also hermitian: $\chi_{ij} = \chi^*_{ji}$

Optical activity

- $\chi_{ij}(\omega,\mathbf{q}) = \chi_{ij}(\omega,0) + \chi^{oa}_{ijk}(\omega) q_k$
- Invariance with respect to time reversal: $\Gamma_{ij}(\tau,\boldsymbol{\xi}) = \Gamma_{ji}(\tau,-\boldsymbol{\xi})$
- which implies $\chi_{ij}(\omega,\mathbf{q}) = \chi_{ji}(\omega,-\mathbf{q})$
- if present, a magnetic field must be inverted as well
- recall that χ_{ij} is also hermitian: $\chi_{ij} = \chi^{*}_{ji}$
- χ^{oa}_{ijk} is purely imaginary and antisymmetric in the first two indexes

Optical activity

- $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ij}(\omega, 0) + \chi^{oa}_{ijk}(\omega) q_k$
- Invariance with respect to time reversal: $\Gamma_{ij}(\tau, \boldsymbol{\xi}) = \Gamma_{ji}(\tau, -\boldsymbol{\xi})$
- which implies $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ji}(\omega, -\mathbf{q})$
- if present, a magnetic field must be inverted as well
- recall that χ_{ij} is also hermitian: $\chi_{ij} = \chi^*_{ji}$
- χ^{oa}_{ijk} is purely imaginary and antisymmetric in the first two indexes
- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk} g_k$ with $g_k = G_{kl} q_l$

Optical activity

- $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ij}(\omega, 0) + \chi^{oa}_{ijk}(\omega) q_k$
- Invariance with respect to time reversal: $\Gamma_{ij}(\tau, \boldsymbol{\xi}) = \Gamma_{ji}(\tau, -\boldsymbol{\xi})$
- which implies $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ji}(\omega, -\mathbf{q})$
- if present, a magnetic field must be inverted as well
- recall that χ_{ij} is also hermitian: $\chi_{ij} = \chi^*_{ji}$
- χ^{oa}_{ijk} is purely imaginary and antisymmetric in the first two indexes
- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk} g_k$ with $g_k = G_{kl} q_l$
- gyration vector g_k depends linearly on the wave vector q_l

Optical activity

- $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ij}(\omega, 0) + \chi^{oa}_{ijk}(\omega) q_k$
- Invariance with respect to time reversal: $\Gamma_{ij}(\tau, \boldsymbol{\xi}) = \Gamma_{ji}(\tau, -\boldsymbol{\xi})$
- which implies $\chi_{ij}(\omega, \mathbf{q}) = \chi_{ji}(\omega, -\mathbf{q})$
- if present, a magnetic field must be inverted as well
- recall that χ_{ij} is also hermitian: $\chi_{ij} = \chi^*_{ji}$
- χ^{oa}_{ijk} is purely imaginary and antisymmetric in the first two indexes
- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk} g_k$ with $g_k = G_{kl} q_l$
- gyration vector g_k depends linearly on the wave vector q_l
- G_{kl} is a rank 2 pseudo-tensor

Digression: Genuine and pseudo tensors

- Coordinate transformation $\mathbf{x} \to \mathbf{x}'$ such that $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$ does not change

Digression: Genuine and pseudo tensors

- Coordinate transformation $\mathbf{x} \to \mathbf{x}'$ such that $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$ does not change
- $x_i' = R_{ij} x_j$ where $RR^\dagger = R^\dagger R = I$

Digression: Genuine and pseudo tensors

- Coordinate transformation $\mathbf{x} \to \mathbf{x}'$ such that $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$ does not change
- $x_i' = R_{ij} x_j$ where $RR^\dagger = R^\dagger R = I$
- $\det RR^\dagger = (\det R)^2 = 1$

Digression: Genuine and pseudo tensors

- Coordinate transformation $\mathbf{x} \to \mathbf{x}'$ such that $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$ does not change
- $x_i' = R_{ij} x_j$ where $RR^\dagger = R^\dagger R = I$
- $\det RR^\dagger = (\det R)^2 = 1$
- $\det R = +1$: proper rotation

Digression: Genuine and pseudo tensors

- Coordinate transformation $\mathbf{x} \rightarrow \mathbf{x}'$ such that $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$ does not change
- $x_i' = R_{ij} x_j$ where $RR^\dagger = R^\dagger R = I$
- $\det RR^\dagger = (\det R)^2 = 1$
- $\det R = +1$: proper rotation
- $\det R = -1$: space inverions and proper rotation

Digression: Genuine and pseudo tensors

- Coordinate transformation $\mathbf{x} \to \mathbf{x}'$ such that $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$ does not change
- $x_i' = R_{ij} x_j$ where $RR^\dagger = R^\dagger R = I$
- $\det RR^\dagger = (\det R)^2 = 1$
- $\det R = +1$: proper rotation
- $\det R = -1$: space inverions and proper rotation
- tensors $T_{ij\dots}$ transform as $T'_{ij\dots} = R_{im} R_{jn} \dots T_{mn\dots}$

Digression: Genuine and pseudo tensors

- Coordinate transformation $\mathbf{x} \to \mathbf{x}'$ such that $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$ does not change
- $x_i' = R_{ij} x_j$ where $RR^\dagger = R^\dagger R = I$
- $\det RR^\dagger = (\det R)^2 = 1$
- $\det R = +1$: proper rotation
- $\det R = -1$: space inverions and proper rotation
- tensors $T_{ij\dots}$ transform as $T'_{ij\dots} = R_{im} R_{jn} \dots T_{mn\dots}$
- they are called genuine tensors

Digression: Genuine and pseudo tensors

- Coordinate transformation $\mathbf{x} \rightarrow \mathbf{x}'$ such that $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$ does not change
- $x_i' = R_{ij} x_j$ where $RR^\dagger = R^\dagger R = I$
- $\det RR^\dagger = (\det R)^2 = 1$
- $\det R = +1$: proper rotation
- $\det R = -1$: space inverions and proper rotation
- tensors $T_{ij\dots}$ transform as $T'_{ij\dots} = R_{im} R_{jn} \dots T_{mn\dots}$
- they are called genuine tensors
- pseudo tensors $P_{ij\dots}$ transform as $P'_{ij\dots} = (\det R)\, R_{im} R_{jn} \dots P_{mn\dots}$

Digression: Genuine and pseudo tensors

- Coordinate transformation $\mathbf{x} \to \mathbf{x}'$ such that $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$ does not change
- $x_i' = R_{ij} x_j$ where $RR^\dagger = R^\dagger R = I$
- $\det RR^\dagger = (\det R)^2 = 1$
- $\det R = +1$: proper rotation
- $\det R = -1$: space inverions and proper rotation
- tensors $T_{ij\dots}$ transform as $T'_{ij\dots} = R_{im} R_{jn} \dots T_{mn\dots}$
- they are called genuine tensors
- pseudo tensors $P_{ij\dots}$ transform as $P'_{ij\dots} = (\det R)\, R_{im} R_{jn} \dots P_{mn\dots}$
- δ_{ij} is a genuine tensor of rank 2

Digression: Genuine and pseudo tensors

- Coordinate transformation $\mathbf{x} \to \mathbf{x}'$ such that $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$ does not change
- $x_i' = R_{ij} x_j$ where $RR^\dagger = R^\dagger R = I$
- $\det RR^\dagger = (\det R)^2 = 1$
- $\det R = +1$: proper rotation
- $\det R = -1$: space inverions and proper rotation
- tensors $T_{ij\dots}$ transform as $T'_{ij\dots} = R_{im} R_{jn} \dots T_{mn\dots}$
- they are called genuine tensors
- pseudo tensors $P_{ij\dots}$ transform as $P'_{ij\dots} = (\det R)\, R_{im} R_{jn} \dots P_{mn\dots}$
- δ_{ij} is a genuine tensor of rank 2
- ϵ_{ijk} is a pseudo tensor of rank 3

Optical activity (ctd.)

- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk}g_k$ with $g_k = G_{kl}q_l$

Optical activity (ctd.)

- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk} g_k$ with $g_k = G_{kl} q_l$
- $\hat{E}_i$ and $\hat{P}_i$ are genuine vectors

Optical activity (ctd.)

- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk}g_k$ with $g_k = G_{kl}q_l$
- $\hat{E}_i$ and $\hat{P}_i$ are genuine vectors
- hence susceptibility χ_{ij} is a genuine tensor of rank 2

Optical activity (ctd.)

- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk}g_k$ with $g_k = G_{kl}q_l$
- $\hat{E}_i$ and $\hat{P}_i$ are genuine vectors
- hence susceptibility χ_{ij} is a genuine tensor of rank 2
- g must be a pseudo vector

Optical activity (ctd.)

- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk}g_k$ with $g_k = G_{kl}q_l$
- $\hat{E}_i$ and $\hat{P}_i$ are genuine vectors
- hence susceptibility χ_{ij} is a genuine tensor of rank 2
- g must be a pseudo vector
- q_i is a genuine vector

Optical activity (ctd.)

- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk}g_k$ with $g_k = G_{kl}q_l$
- $\hat{E}_i$ and $\hat{P}_i$ are genuine vectors
- hence susceptibility χ_{ij} is a genuine tensor of rank 2
- g must be a pseudo vector
- q_i is a genuine vector
- G_{kl} therefore is a rank 2 pseudo tensor

Optical activity (ctd.)

- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk}g_k$ with $g_k = G_{kl}q_l$
- $\hat{E}_i$ and $\hat{P}_i$ are genuine vectors
- hence susceptibility χ_{ij} is a genuine tensor of rank 2
- g must be a pseudo vector
- q_i is a genuine vector
- G_{kl} therefore is a rank 2 pseudo tensor
- Only materials which have no mirror symmetry show optical activity

Optical activity (ctd.)

- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk} g_k$ with $g_k = G_{kl} q_l$
- $\hat{E}_i$ and $\hat{P}_i$ are genuine vectors
- hence susceptibility χ_{ij} is a genuine tensor of rank 2
- g must be a pseudo vector
- q_i is a genuine vector
- G_{kl} therefore is a rank 2 pseudo tensor
- Only materials which have no mirror symmetry show optical activity
- Material must distinguish between left and right handed

Optical activity (ctd.)

- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk} g_k$ with $g_k = G_{kl} q_l$
- $\hat{E}_i$ and $\hat{P}_i$ are genuine vectors
- hence susceptibility χ_{ij} is a genuine tensor of rank 2
- g must be a pseudo vector
- q_i is a genuine vector
- G_{kl} therefore is a rank 2 pseudo tensor
- Only materials which have no mirror symmetry show optical activity
- Material must distinguish between left and right handed
- dextrose, quartz, ...

Optical activity (ctd.)

- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk}g_k$ with $g_k = G_{kl}q_l$
- $\hat{E}_i$ and $\hat{P}_i$ are genuine vectors
- hence susceptibility χ_{ij} is a genuine tensor of rank 2
- g must be a pseudo vector
- q_i is a genuine vector
- G_{kl} therefore is a rank 2 pseudo tensor
- Only materials which have no mirror symmetry show optical activity
- Material must distinguish between left and right handed
- dextrose, quartz, ...
- Optically active materials cause a rotation of the polarization proportional to the sample thickness

Optical activity (ctd.)

- $\Delta\chi^{oa}_{ij} = i\epsilon_{ijk} g_k$ with $g_k = G_{kl} q_l$
- $\hat{E}_i$ and $\hat{P}_i$ are genuine vectors
- hence susceptibility χ_{ij} is a genuine tensor of rank 2
- g must be a pseudo vector
- q_i is a genuine vector
- G_{kl} therefore is a rank 2 pseudo tensor
- Only materials which have no mirror symmetry show optical activity
- Material must distinguish between left and right handed
- dextrose, quartz, ...
- Optically active materials cause a rotation of the polarization proportional to the sample thickness
- The effect is reversible, as contrasted with the Faraday effect

Dextrose

- dextrose is naturally produced sugar

Dextrose

- dextrose is naturally produced sugar
- glucose (sugar) from greek $\gamma\lambda\upsilon\kappa\upsilon\varsigma$=glycys=sweet

Dextrose

- dextrose is naturally produced sugar
- glucose (sugar) from greek $\gamma\lambda\upsilon\kappa\upsilon\varsigma$=glycys=sweet
- dextrose from latin dexter=right

Dextrose

- dextrose is naturally produced sugar
- glucose (sugar) from greek $\gamma\lambda\upsilon\kappa\upsilon\varsigma$=glycys=sweet
- dextrose from latin dexter=right
- artificially produced sugar does not show left/right-handedness

Dextrose

- dextrose is naturally produced sugar
- glucose (sugar) from greek $\gamma\lambda\upsilon\kappa\upsilon\varsigma$=glycys=sweet
- dextrose from latin dexter=right
- artificially produced sugar does not show left/right-handedness
- biologically produces sugar (dextrose) is optically effective

Dextrose

- dextrose is naturally produced sugar
- glucose (sugar) from greek $\gamma\lambda\upsilon\kappa\upsilon\varsigma$=glycys=sweet
- dextrose from latin dexter=right
- artificially produced sugar does not show left/right-handedness
- biologically produces sugar (dextrose) is optically effective
- quantum mechanically the state $\phi_L + \phi_R$ has lowest energy

Dextrose

- dextrose is naturally produced sugar
- glucose (sugar) from greek $\gamma\lambda\upsilon\kappa\upsilon\varsigma$=glycys=sweet
- dextrose from latin dexter=right
- artificially produced sugar does not show left/right-handedness
- biologically produces sugar (dextrose) is optically effective
- quantum mechanically the state $\phi_L + \phi_R$ has lowest energy
- however, transition probability is extremely small

Dextrose

- dextrose is naturally produced sugar
- glucose (sugar) from greek $\gamma\lambda\upsilon\kappa\upsilon\varsigma$=glycys=sweet
- dextrose from latin dexter=right
- artificially produced sugar does not show left/right-handedness
- biologically produces sugar (dextrose) is optically effective
- quantum mechanically the state $\phi_L + \phi_R$ has lowest energy
- however, transition probability is extremely small
- molecules ϕ_R as produced by the plant live a very, very long time (comparable with the age of the universe)

Dextrose

- dextrose is naturally produced sugar
- glucose (sugar) from greek $\gamma\lambda\upsilon\kappa\upsilon\varsigma$=glycys=sweet
- dextrose from latin dexter=right
- artificially produced sugar does not show left/right-handedness
- biologically produces sugar (dextrose) is optically effective
- quantum mechanically the state $\phi_L + \phi_R$ has lowest energy
- however, transition probability is extremely small
- molecules ϕ_R as produced by the plant live a very, very long time (comparable with the age of the universe)
- Nature has no tendency to prefer left to right handedness

Dextrose

- dextrose is naturally produced sugar
- glucose (sugar) from greek $\gamma\lambda\upsilon\kappa\upsilon\varsigma$=glycys=sweet
- dextrose from latin dexter=right
- artificially produced sugar does not show left/right-handedness
- biologically produces sugar (dextrose) is optically effective
- quantum mechanically the state $\phi_L + \phi_R$ has lowest energy
- however, transition probability is extremely small
- molecules ϕ_R as produced by the plant live a very, very long time (comparable with the age of the universe)
- Nature has no tendency to prefer left to right handedness
- Question: Are all sugar producing plants copies of the first plant, which randomly decided between left and right?

Quartz

- Quartz is silicon dioxide, SiO_2

Quartz

- Quartz is silicon dioxide, SiO_2
- the most common mineral on earth

Quartz

- Quartz is silicon dioxide, SiO_2
- the most common mineral on earth
- just think of sand

Quartz

- Quartz is silicon dioxide, SiO_2
- the most common mineral on earth
- just think of sand
- single crystals are either left or right optically active

Quartz

- Quartz is silicon dioxide, SiO_2
- the most common mineral on earth
- just think of sand
- single crystals are either left or right optically active
- however, twins are also found

Quartz

- Quartz is silicon dioxide, SiO_2
- the most common mineral on earth
- just think of sand
- single crystals are either left or right optically active
- however, twins are also found
- world wide distribution is very close to 1:1 for left:right crystals

Quartz

- Quartz is silicon dioxide, SiO_2
- the most common mineral on earth
- just think of sand
- single crystals are either left or right optically active
- however, twins are also found
- world wide distribution is very close to 1:1 for left:right crystals
- there seems to be no preference of nature for left or right

Quartz

- Quartz is silicon dioxide, SiO_2
- the most common mineral on earth
- just think of sand
- single crystals are either left or right optically active
- however, twins are also found
- world wide distribution is very close to 1:1 for left:right crystals
- there seems to be no preference of nature for left or right
- although parity is not a symmetry of nature...

Quartz

- Quartz is silicon dioxide, SiO_2
- the most common mineral on earth
- just think of sand
- single crystals are either left or right optically active
- however, twins are also found
- world wide distribution is very close to 1:1 for left:right crystals
- there seems to be no preference of nature for left or right
- although parity is not a symmetry of nature. . .
- TCP is

Quartz

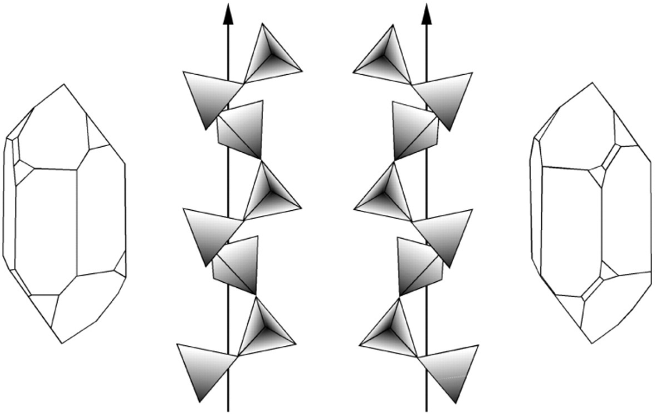
- Quartz is silicon dioxide, SiO_2
- the most common mineral on earth
- just think of sand
- single crystals are either left or right optically active
- however, twins are also found
- world wide distribution is very close to 1:1 for left:right crystals
- there seems to be no preference of nature for left or right
- although parity is not a symmetry of nature...
- TCP is
- P alone not

Quartz

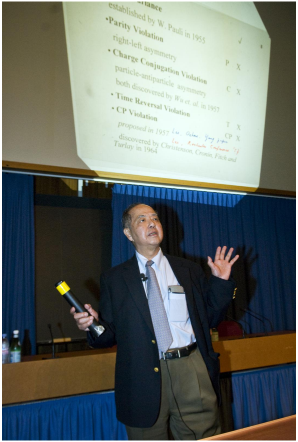
- Quartz is silicon dioxide, SiO_2
- the most common mineral on earth
- just think of sand
- single crystals are either left or right optically active
- however, twins are also found
- world wide distribution is very close to 1:1 for left:right crystals
- there seems to be no preference of nature for left or right
- although parity is not a symmetry of nature. . .
- TCP is
- P alone not
- but only in weak interactions

Quartz

- Quartz is silicon dioxide, SiO_2
- the most common mineral on earth
- just think of sand
- single crystals are either left or right optically active
- however, twins are also found
- world wide distribution is very close to 1:1 for left:right crystals
- there seems to be no preference of nature for left or right
- although parity is not a symmetry of nature. . .
- TCP is
- P alone not
- but only in weak interactions
- read about Lee and Yang: http://ccreweb.org/documents/parity/parity.html



l-quartz and d-quartz



Lee Tsung Dao, Nobel Prize 1957

Yang Chen Ning, Nobel Prize 1957