Surface Plasma Polaritons

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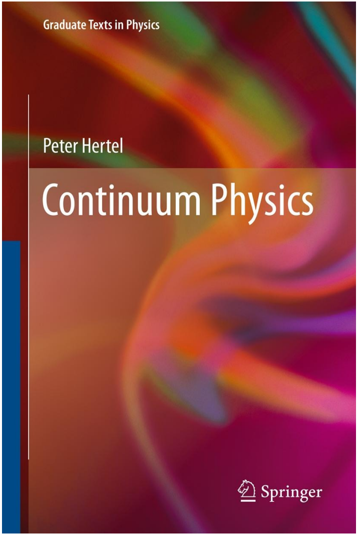
Overview

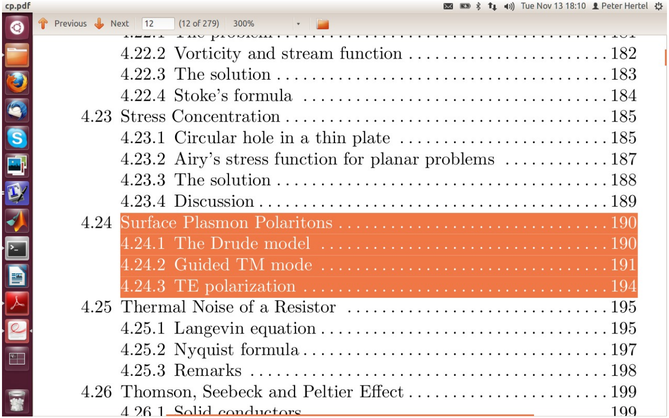
- Introduction
- Drude model
- Guided TM mode
- Excitation of SPS

Introduction

- Surface Plasmon Polaritons - SPP
- are guided by the discontinuity of a surface
- they require a plasma of charges - metals
- this will be polarized
- Polaritons, because they are quasi-particles, just like phonons
- The latter aspect is ignored here
- We have to study Maxwell's equations in a metal
- A noble metal like gold or silver is best



The topic is discussed in this new book



The index entry (screenshot)

Drude model

- Consider a typical electron of effective mass m and charge $q = -e$
- Its deviation from its average position is x
- It is exposed to an electric field E
- The equation of motion is $m\{\ddot{x} + \mathit{\Gamma}\dot{x} + \mathit{\Omega}^2 x\} = q\,E(t)$
- Fourier transform it: $f(t) = \int \frac{\mathrm{d}\omega}{2\pi}\,\tilde{f}(\omega)\,\mathrm{e}^{-\mathrm{i}\omega t}$
- The equation of motion becomes $m\{-\omega^2 - \mathrm{i}\mathit{\Gamma}\omega + \mathit{\Omega}^2\}\,\tilde{x} = q\tilde{E}$
- The solution of which is $\tilde{x}(\omega) = \frac{q}{m}\frac{\tilde{E}(\omega)}{-\omega^2 - \mathrm{i}\mathit{\Gamma}\omega + \mathit{\Omega}^2}$

Paul Drude, German physicist, 1863-1906; Leipzig, Gießen, Berlin; shot himself for unknown reasons after being elected as Member of the Prussian Academy of Sciences.

Drude model ctd.

- Recall

$$\tilde{x}(\omega) = \frac{q}{m} \frac{\tilde{E}(\omega)}{-\omega^2 - \mathrm{i}\mathit{\Gamma}\omega + \mathit{\Omega}^2}$$

- $\tilde{p} = q\tilde{x}$ is the oscillating dipole moment
- There are N typical electrons per unit volume
- The polarizaton of the material therefore is

$$\tilde{P}(\omega) = Nq\tilde{x}(\omega) = \frac{Nq^2}{m} \frac{\tilde{E}(\omega)}{-\omega^2 - \mathrm{i}\mathit{\Gamma}\omega + \mathit{\Omega}^2}$$

- or

$$\tilde{P}\omega) = \epsilon_0 \chi(\omega) \tilde{E}(\omega)$$

- with susceptibility

$$\chi(\omega) = \frac{Nq^2}{m\epsilon_0} \frac{1}{-\omega^2 - \mathrm{i}\mathit{\Gamma}\omega + \mathit{\Omega}^2}$$

Dielectric medium

- Recall

$$\chi(\omega)=\frac{Nq^2}{m\epsilon_0}\frac{1}{-\omega^2-\mathrm{i}\Gamma\omega+\Omega^2}$$

- The electrons are elastically bound to their ions
- spring constant $m\Omega^2>0$
- static susceptibility

$$\chi(0)=\frac{Nq^2}{m\epsilon_0\Omega^2}$$

- $\chi(\omega)\to 0$ for $\omega\to\infty$
- $\chi(\omega)=\chi'(\omega)+\mathrm{i}\chi''(\omega)$ is complex!
- Kramers-Kronig relations

$$\chi'(\omega)=\int_0^\infty\frac{\mathrm{d}u}{\pi}\,\frac{2u\chi''(u)}{u^2-\omega^2}$$

- reflecting causality

Conducting medium

- Recall

$$\chi(\omega) = \frac{Nq^2}{m\epsilon_0} \frac{1}{-\omega^2 - \mathrm{i}\mathit{\Gamma}\omega + \mathit{\Omega}^2}$$

- Charges are a plasma
- not bound to ions
- $\mathit{\Omega} = 0$
- electric alternating current $J = \dot{P}$ or $\tilde{J} = -\mathrm{i}\omega\tilde{P}$
- Drude model

$$\tilde{J} = \frac{Nq^2}{m\epsilon_0} \frac{\tilde{E}}{\mathit{\Gamma} + \mathrm{i}\omega}$$

- Ohm's law

$$\tilde{J}(0) = \sigma\tilde{E}(0)$$

- conductivity is

$$\sigma = \frac{Nq^2}{m\epsilon_0\mathit{\Gamma}}$$

Permittivity of metals

- The permittivity of a metal my be written as

$$\epsilon(\omega) = \epsilon_\infty - \frac{\omega_\mathrm{P}^2}{\omega^2 + \mathrm{i}\omega\Gamma}$$

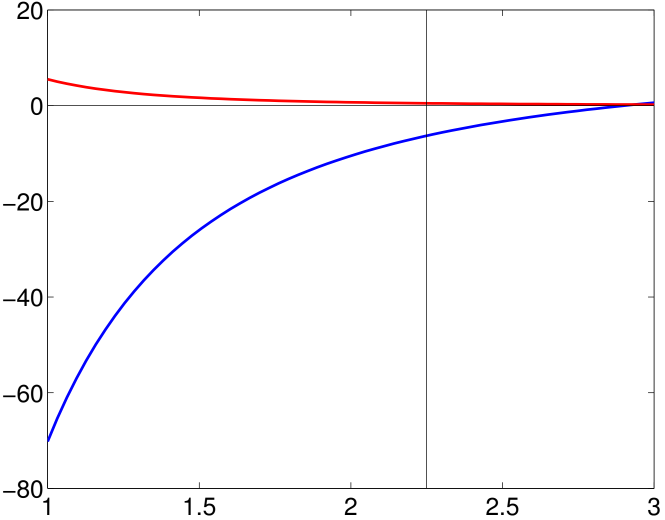
- The so called plasma frequency ω_P is given by

$$\omega_\mathrm{P}^2 = \frac{Nq^2}{m\epsilon_0}$$

- where N denotes the number of charge carriers per unit volume and m their effective mass
- ϵ_∞ describes contributions from other sources of polarization, much much above the plasma frequency

Gold

- Johnson and Christy have thoroughly investigated the optical properties of the noble metals
- here the results for gold
- $\epsilon_\infty = 9.50$
- $\hbar\omega_\mathrm{P} = 8.95$ eV
- $\hbar\mathit{\Gamma} = 0.069$ eV
- The Drude model fits well for energies $\hbar\omega$ less than 2.25 eV
- For higher photon energies, inter-band transitions occur which cannot be described by the Drude model



Real part (blue) and imaginary part (red) of the permittivity of gold plotted versus photon energy in eV. Experimental data fit well left to the vertical line.

Surface TM mode

- x is coordinate perpendicular to the metal surface
- $x > 0$ is the cover (subscript c)
- $x < 0$ the metal (subscript m)
- Surface wave propagates in z direction
- All fields are of the form
  $$F(t,x,y,z) = F(x)\,\mathrm{e}^{\,\mathrm{i}\beta z}\;\mathrm{e}^{\,-\mathrm{i}\omega t}$$
- TM mode ansatz
  $$\boldsymbol{H} = \begin{pmatrix} 0 \\ H \\ 0 \end{pmatrix} \text{ and } \epsilon\,\boldsymbol{E} = \frac{1}{\omega\epsilon_0}\begin{pmatrix} \beta H \\ 0 \\ \mathrm{i}H' \end{pmatrix}$$
- $\epsilon E_x, E_y, E_z, H_x, H_y, H_z$ must be continuous at $x = 0$
- i. e. H and H'/ϵ have to be continuous

Surface TM mode (ctd.)

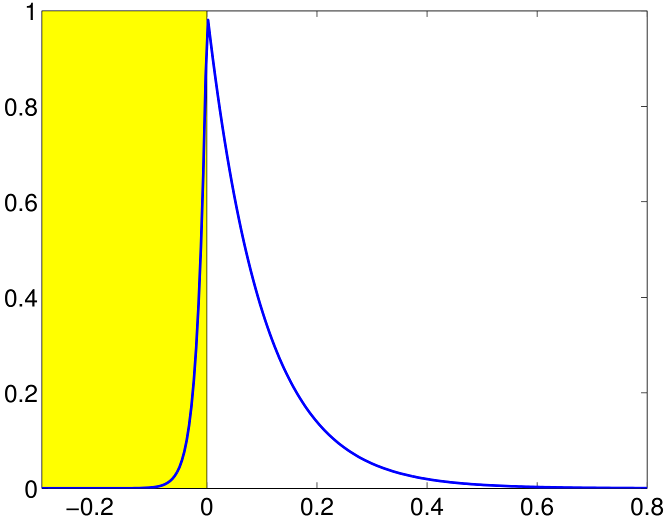
- The TM mode equation reads
$$\left\{\epsilon(x)\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{\epsilon(x)}\frac{\mathrm{d}}{\mathrm{d}x}+k_0^2\epsilon(x)\right\}H=\beta^2 H$$
- For a piece-wise constant permittivity this simplifies to
$$H''+k_0^2\epsilon(x)H=\beta^2 H$$
- Define $\kappa_{\mathrm{c}}=\sqrt{\beta^2-k_0^2\epsilon_{\mathrm{c}}}$ and $\kappa_{\mathrm{m}}=\sqrt{\beta^2-k_0^2\epsilon_{\mathrm{m}}}$
- square root of complex numbers always with positive real part!
- solution in the cover region $x>0$
$$H(x)=\mathrm{e}^{-\kappa_{\mathrm{c}}x}$$
- solution in the metal region $x<0$
$$H(x)=\mathrm{e}^{+\kappa_{\mathrm{m}}x}$$
- solution is already continuous

Surface TM mode (ctd.)

- However, H'/ϵ must also be continuous
- which requires
$$\frac{-\kappa_\mathrm{c}}{\epsilon_\mathrm{c}} = \frac{+\kappa_\mathrm{m}}{\epsilon_\mathrm{m}}$$
- Squaring this yields
$$\beta^2 = k_0^2 \, \frac{\epsilon_\mathrm{m}\,\epsilon_\mathrm{c}}{\epsilon_\mathrm{m} + \epsilon_\mathrm{c}}$$
- If ϵ_m is real, negative and sufficiently large, the continuity conditions can be met
- and β is real
- If ϵ_m has a small imaginary contribution, then β as well,
- which means wave attenuation (damping)
- One TM , no TE surface mode possible

A case study

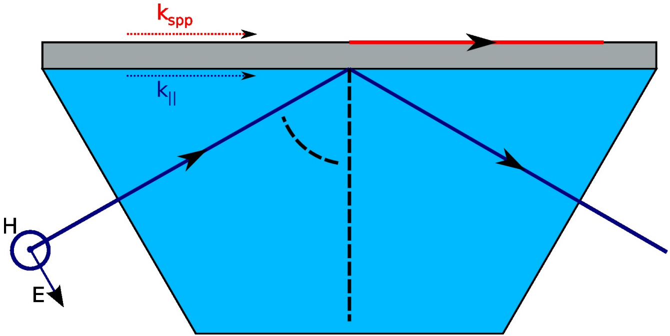
- $\hbar\omega = 2.25$
- i. e. $\lambda =$ 551.3 nm
- $k_0 =$ 11.40 µm^{-1}
- $\epsilon_\mathrm{c} =$ 1.000
- $\epsilon^\mathrm{m} =$ (-6.308 + 0.4848 i)
- $\kappa_\mathrm{c} =$ (4.931 + 0.2247 i) µm^{-1}
- $\kappa_\mathrm{m} =$ (32.21 - 0.9730 i) µm^{-1}
- $\beta =$ (12.42 + 0.08925 i) µm^{-1}
- $\ell = 1/2\,\mathrm{Im}\,\beta =$ 5.602 µm



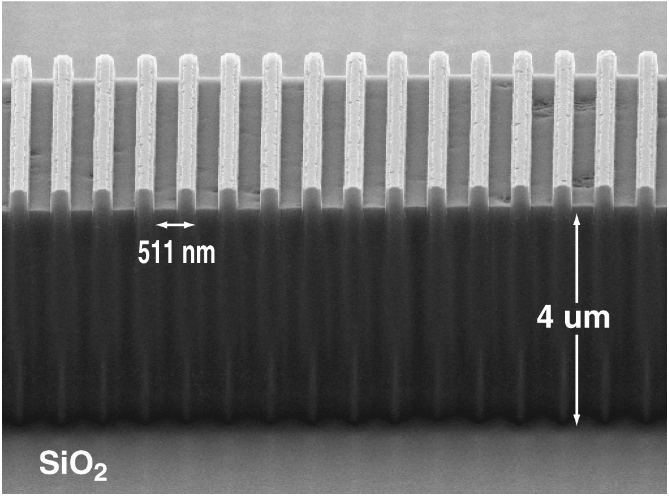
Gold is covered by air. The intensity $|H(x)|^2$ is plotted vs. the distance (in μm) from the surface The mode penetrates ca. 16 nm into the metal. Light wavelength is 551.3 nm.

Exciting SSPs

- β is largen than k_0
- But k_0 in vacuum becomes nk_0 in a medium, where n is the refractive index
- Prism, $\beta = nk_0 \sin\theta$
- See sketch
- Similar effect may be obtained with the aid of a bragg grating
- Third possibility is
- irradiating the metal with electron
- which will scatter
- and some have the required momentum $\hbar k_z$
- and will trigger plasma waves of the right wavelength
- which are the source of a surface TM mode



The z-component of the wave vector in the prism must match the propagation constant β of the surface-guided TM mode



The grating produces wave vectors mK where m is an integer. They are added to the light wave vector. The grid spacing must be chosen such that $k_0 \cos\theta + mK = \beta$