Peter Hertel

Fundamenta particles

Spin and statistics

Many particles

Gibbs state

Fermions

Zero temperature

White dwarfs and neutron stars

Bosons

Black body radiation

Bose-Einstein condensation

Fermions, Bosons and their Statistics

Peter Hertel

University of Osnabrück, Germany

Lecture presented at APS, Nankai University, China

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March/April 2011

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Structure fermions

charge	-1	-2/3	-1/3	0	+1/3	+2/3	+1
generation 1	<i>e</i> ⁻	ū	d	$\nu_e, \bar{\nu}_e$	ā	и	e ⁺
generation 2	μ^{-}	ī	5	$ u_{\mu}, ar{ u}_{\mu}$	5	с	μ^+
generation 3	τ^{-}	ī	Ь	$ u_{ au}, ar{ u}_{ au}$	Б	t	τ^+

• all structure particles have spin 1/2

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Structure fermions

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- all structure particles have spin 1/2
- proton p=(uud) and neutron n=(udd)

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- proton p=(uud) and neutron n=(udd)
- both have spin 1/2
- excited states $\Delta^{++}=(uuu)^*$, $\Delta^+=(uud)^*$ etc.

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- all structure particles have spin 1/2
- proton p=(uud) and neutron n=(udd)
- both have spin 1/2
- excited states $\Delta^{++}=(uuu)^*$, $\Delta^+=(uud)^*$ etc.
- they have spin 3/2
- p is stable, n \rightarrow p+e^++ $\bar{\nu}$ is allowed

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Exchange bosons

W^{-}	$\bar{\nu}_e e^- \bar{\nu}_\mu \mu^- \bar{\nu}_ au au^-$	ūd īcs ītb
γ, Z^{0}	$e^+e^- \mu^+\mu^- au^+ au^- (ar u_e u_e \ ar u_\mu u_\mu \ ar u_ au u_ au)$	dd ūu ss cc bb tt
W^+	$e^+ u_e\;\mu^+ u_\mu\; au^+ u_ au$	du sc bt

- W⁺, γ , Z⁰, W⁻ mediate electro-weak interaction

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$\begin{array}{c|cccc} W^{-} & \bar{\nu}_{e}e^{-} \; \bar{\nu}_{\mu}\mu^{-} \; \bar{\nu}_{\tau}\tau^{-} & \bar{u}d\; \bar{c}s\; \bar{t}b \\ \hline \gamma, Z^{0} & e^{+}e^{-} \; \mu^{+}\mu^{-} \; \tau^{+}\tau^{-} \; (\bar{\nu}_{e}\nu_{e}\; \bar{\nu}_{\mu}\nu_{\mu}\; \bar{\nu}_{\tau}\nu_{\tau}) & \bar{d}d\; \bar{u}u\; \bar{s}s\; \bar{c}c\; \bar{b}b\; \bar{t}t \\ \hline W^{+} & e^{+}\nu_{e}\; \mu^{+}\nu_{\mu}\; \tau^{+}\nu_{\tau} & \bar{d}u\; \bar{s}c\; \bar{b}t \end{array}$

Exchange bosons

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the photon couples to charged particles only

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W-	$\bar{ u}_e e^- \bar{ u}_\mu \mu^- \bar{ u}_ au au^-$	ūd īcs ītb
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• W⁺, γ , Z⁰, W⁻ mediate electro-weak interaction

- the photon couples to charged particles only
- · there are also gluons which mediate strong interactions

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• W⁺, γ , Z⁰, W⁻ mediate electro-weak interaction

the photon couples to charged particles only

- there are also gluons which mediate strong interactions
- e.g. n=(udd) \rightarrow (uud)+W⁻ \rightarrow p+ $\bar{\nu}_e$ +e⁻

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the photon couples to charged particles only

- there are also gluons which mediate strong interactions
- e.g. n=(udd) \rightarrow (uud)+W⁻ \rightarrow p+ $\bar{\nu}_e$ +e⁻
- is there also a graviton G ?

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• W⁺, γ , Z⁰, W⁻ mediate electro-weak interaction

the photon couples to charged particles only

- there are also gluons which mediate strong interactions
- e.g. n=(udd) \rightarrow (uud)+W⁻ \rightarrow p+ $\bar{\nu}_e$ +e⁻
- is there also a graviton G ?
- is there also a Higgs boson H ?

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The Aleph detector (opened) at Cern

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Spin 1/2 fermions

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• massive structure fermion (quarks, electrons) have spin 1/2

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Spin 1/2 fermions

- massive structure fermion (quarks, electrons) have spin 1/2
- or helicity 1/2 and -1/2

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Spin 1/2 fermions

- massive structure fermion (quarks, electrons) have spin $1/2\,$
- or helicity 1/2 and -1/2
- massless structure fermions (neutrinos) have either helicity 1/2 \underline{or} -1/2

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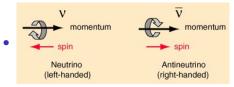
Black body radiation

Bose-Einstein condensation

Spin 1/2 fermions

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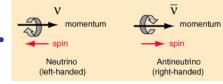
Bose-Einstein condensation

Spin 1/2 fermions

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this explains parity violation

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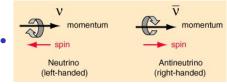
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Spin 1/2 fermions

- massive structure fermion (quarks, electrons) have spin 1/2
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- this explains parity violation
- spin and statistics are related

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Spin and statisics

• rotation of a particle with spin s is described by $\psi_{\rm R} = e^{\frac{i}{\hbar} \alpha \cdot \mathbf{S}} \psi$

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Spin and statisics

- rotation of a particle with spin s is described by $\psi_{\rm R} = e^{\frac{i}{\hbar} \alpha \cdot \mathbf{S}} \psi$
- a 360° rotation yields $\bar{\psi} = e^{2\pi i s} \psi = \phi_R \psi$

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- $\phi_{\rm R} = +1$ for integer spin (bosons), and -1 for half-integer spin (fermions)

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- exchanging two identicle particles must not change $|\psi|^2$ $\psi(\sigma_1 \mathbf{x}_1, \sigma_2 \mathbf{x}_2) = \phi_{\mathbf{X}} \psi(\sigma_2 \mathbf{x}_2, \sigma_1 \mathbf{x}_1)$

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- spin-statistics theorem says $\phi_{
 m R} = \phi_{
 m X}$
- for fermions: $\psi(\sigma_1 \mathbf{x}_1, \sigma_2 \mathbf{x}_2) = -\psi(\sigma_2 \mathbf{x}_2, \sigma_1 \mathbf{x}_1)$

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- Pauli exclusion principle

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Wolfgang Pauli, Austrian/Swiss Physicist, Nobel prize 1945

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Arbitrary number of particles

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Arbitrary number of particles

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•
$$(A, A) = 0, (A^*, A^*) = 0, (A, A^*) = I$$

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- fermions: $(X, Y) = \{X, Y\} = XY + YX$, the anti-commutator

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- *N* = *A***A* is number operator

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- create n particles

$$\psi_n = \frac{1}{\sqrt{n!}} \left(A^* \right)^n \Omega$$

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$$N\psi_n = n\psi_r$$

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- create n particles

$$\psi_n = \frac{1}{\sqrt{n!}} \, (A^*)^n \, \Omega$$

•
$$N\psi_n = n\psi_n$$

• bosons: n = 0, 1, 2, ...

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- create n particles

$$\psi_n = \frac{1}{\sqrt{n!}} \, (A^*)^n \, \Omega$$

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$$N\psi_n = n\psi_n$$

- bosons: n = 0, 1, 2, ...
- fermions: n = 0, 1

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Many states

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• *j* labels one-particle states

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Many particles

Gibbs state

Fermions

Zero temperature

White dwarfs and neutron stars

Bosons

Black body radiation

Bose-Einstein condensation

• *j* labels one-particle states

• creators A_i^* and annihilators A_j

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- *j* labels one-particle states
- creators A_i^* and annihilators A_i
- $(A_j, A_k) = (A_j^*, A_k^*) = 0$ and $(A_j, A_k^*) = \delta_{jk}$

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- $N_j = A_j^* A_j$ has eigenvalues 0 and 1, i.e.

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- $A_j^* A_k^* \Omega = -A_k^* A_j^* \Omega$
- $N_j = A_j^* A_j$ has eigenvalues 0 and 1, i.e.
- a state is occupied at most once
- note that [N_j, N_k] = 0

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Energy

• $H = \sum_{i} \epsilon_{j} A_{i}^{*} A_{j} + \sum_{jklm} V_{jklm} A_{j}^{*} A_{k}^{*} A_{l} A_{m}$

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• $H = \sum_{i} \epsilon_{j} A_{i}^{*} A_{j} + \sum_{jklm} V_{jklm} A_{i}^{*} A_{k}^{*} A_{l} A_{m}$

• if interaction term can be neglected

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• $H = \sum_{j} \epsilon_{j} A_{j}^{*} A_{j} + \sum_{jklm} V_{jklm} A_{j}^{*} A_{k}^{*} A_{l} A_{m}$

• if interaction term can be neglected

•
$$H = \sum_j \epsilon_j N_j$$

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• $H = \sum_j \epsilon_j A_j^* A_j + \sum_{jklm} V_{jklm} A_j^* A_k^* A_l A_m$

- if interaction term can be neglected
- $H = \sum_j \epsilon_j N_j$
- examples are quasi-free electrons (hopping model) or phonons

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• $H = \sum_j \epsilon_j A_j^* A_j + \sum_{jklm} V_{jklm} A_j^* A_k^* A_l A_m$

• if interaction term can be neglected

•
$$H = \sum_j \epsilon_j N_j$$

- examples are quasi-free electrons (hopping model) or phonons
- $N = \sum_{j} N_{j}$ is the particle number operator

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•
$$H = \sum_j \epsilon_j A_j^* A_j + \sum_{jklm} V_{jklm} A_j^* A_k^* A_l A_m$$

• if interaction term can be neglected

•
$$H = \sum_{j} \epsilon_{j} N_{j}$$

- examples are quasi-free electrons (hopping model) or phonons
- $N = \sum_{j} N_{j}$ is the particle number operator
- note that the number of particles in a system is not fixed (open system)

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Free energy

• The equilibrium (Gibbs) state of an open system is $G = e^{(F - H + \mu N)/k_{\rm B}T}$

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- The equilibrium (Gibbs) state of an open system is $G = e^{(F H + \mu N)/k_{\rm B}T}$
- free energy F defined by tr G = 1

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• The equilibrium (Gibbs) state of an open system is $G = e^{(F - H + \mu N)/k_{B}T}$

- free energy F defined by tr G = 1
- which gives

$$F = -k_{
m B}T \ln$$
 tr $e^{\left(\mu N - H
ight)/k_{
m B}T}$

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• The equilibrium (Gibbs) state of an open system is $G = e^{(F - H + \mu N)/k_{\rm B}T}$

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$${\it F}=-{\it k}_{
m B}{\it T}$$
 In tr $e^{\left(\mu N-H
ight)/{\it k}_{
m B}{\it T}}$

• temperature T defined by tr $GH = U = \overline{H}$

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- free energy F defined by tr G = 1
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$${\it F}=-{\sf k}_{\sf B}{\it T}$$
 In tr $e^{\left(\mu {\it N}-{\it H}
ight)/{\sf k}_{\sf B}{\it T}}$

- temperature T defined by tr $GH = U = \overline{H}$
- chemical potential μ defined by tr $GN = \bar{N}$

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• The equilibrium (Gibbs) state of an open system is $G = e^{(F - H + \mu N)/k_{\rm B}T}$

- free energy F defined by tr G = 1
- which gives

$${m F}=-{f k}_{
m B}{m T}$$
 In tr $e^{ig(\mu {m N}-{m H}ig)/{f k}_{
m B}{m T}}$

- temperature T defined by tr $GH = U = \overline{H}$
- chemical potential μ defined by tr ${\it GN}=ar{\it N}$
- generalization to more than one species of particles is obvious



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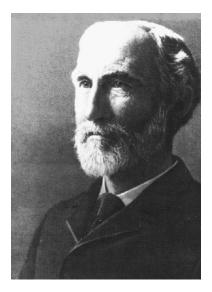
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Josiah Willard Gibbs, US American physicist, 1839-1903

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Fermi-Dirac statistics

• because particle number operators commute $e^{(\mu N - H)/k_{B}T} = \prod_{i} e^{(\mu - \epsilon_{j})N_{j}/k_{B}T}$

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Fermi-Dirac statistics

• because particle number operators commute $e^{(\mu N - H)/k_{B}T} = \prod_{i} e^{(\mu - \epsilon_{j})N_{j}/k_{B}T}$

• work out the trace, i. e. sum over eigenvalues n = 0, 1tr $e^{(\mu N - H)/k_{\rm B}T} = \prod_{j} \left(1 + e^{(\mu - \epsilon_j)/k_{\rm B}T}\right)$

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Fermi-Dirac statistics

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- work out the trace, i. e. sum over eigenvalues n = 0, 1tr $e^{(\mu N - H)/k_{\rm B}T} = \prod_{j} \left(1 + e^{(\mu - \epsilon_j)/k_{\rm B}T}\right)$

$$F = -k_{\rm B}T \sum_{j} \ln\left(1 + e^{(\mu - \epsilon_j)/k_{\rm B}T}\right)$$

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Fermi-Dirac statistics

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- free energy

$$F = -k_{\rm B}T \sum_{j} \ln\left(1 + e^{(\mu - \epsilon_j)/k_{\rm B}T}\right)$$

• in $[\epsilon, \epsilon + \mathrm{d}\epsilon]$ there are $V \, z(\epsilon) \, \mathrm{d}\epsilon$ single particle states

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Fermi-Dirac statistics

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- free energy

$$F = -k_{\rm B}T \sum_{j} \ln\left(1 + e^{(\mu - \epsilon_j)/k_{\rm B}T}\right)$$

- in $[\epsilon, \epsilon + d\epsilon]$ there are $V z(\epsilon) d\epsilon$ single particle states
- one may write

$$F = -k_{B}TV \int d\epsilon \ z(\epsilon) \ln \left(1 + e^{(\mu - \epsilon)/k_{B}T}\right)$$

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Enrico Fermi, Italian/USA physicist, 1901-1954

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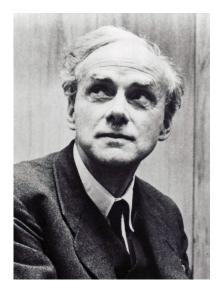
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Paul Dirac, Britisch physicist, 1902-1984

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Non-relativistic fermions

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• sum over states (factor 2 for spin) $\sum_{j} = 2 \int \frac{\mathrm{d}^{3} x \, \mathrm{d}^{3} p}{(2\pi\hbar)^{3}}$

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Non-relativistic fermions

• sum over states (factor 2 for spin) $\sum_{j} = 2 \int \frac{d^{3}x d^{3}p}{(2\pi\hbar)^{3}}$ • $\epsilon = p^{2}/2m$

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Non-relativistic fermions

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• sum over states (factor 2 for spin) $\sum_{j} = 2 \int \frac{d^{3}x d^{3}p}{(2\pi\hbar)^{3}}$ • $\epsilon = p^{2}/2m$

$$\sum_{j} = V \int d\epsilon z(\epsilon)$$
$$= V \frac{1}{\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \int d\epsilon \sqrt{\epsilon}$$

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Particle density and pressure

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recall

$$\mathcal{F} = -k_{\mathsf{B}}TV\int\mathrm{d}\epsilon\,z(\epsilon)\,\ln\,\left(1+\,e^{\,\left(\mu\,-\,\epsilon
ight)/\mathsf{k}_{\mathsf{B}}T\,
ight)$$

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$$F = -k_{\rm B}TV \int \mathrm{d}\epsilon \, z(\epsilon) \, \ln \left(1 + \, e^{\left(\mu - \epsilon\right)/k_{\rm B}T}\right)$$

• particle density is

$$T, \mu) = -\frac{1}{V} \frac{\partial F}{\partial \mu}$$

= $\int d\epsilon z(\epsilon) \frac{1}{e^{(\epsilon - \mu)/k_{\rm B}T} + 1}$

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Particle density and pressure

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$$F = -k_{B}TV \int d\epsilon \, z(\epsilon) \, \ln \left(1 + \, e^{\left(\mu - \epsilon\right)/k_{B}T}\right)$$

• particle density is

$$n(T,\mu) = -\frac{1}{V} \frac{\partial F}{\partial \mu}$$
$$= \int d\epsilon \, z(\epsilon) \, \frac{1}{e^{(\epsilon - \mu)/k_{\rm B}T} + 1}$$

• pressure is

$$p = -\frac{\partial F}{\partial V}$$

= $k_{B}T \int d\epsilon z(\epsilon) \ln \left(1 + e^{(\mu - \epsilon)/k_{B}T}\right)$
= $\frac{2}{3} \int d\epsilon z(\epsilon) \frac{\epsilon}{e^{(\epsilon - \mu)/k_{B}T} + 1}$

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$$\frac{\text{if } T \to 0}{\frac{1}{e^{(\epsilon - \mu)/k_{\text{B}}T} + 1}} = \theta(\mu - \epsilon)$$

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Zero temperature

if
$$T
ightarrow 0$$

 $rac{1}{e^{(\epsilon-\mu)/\mathsf{k}_{\mathsf{B}}T}+1} = heta(\mu-\epsilon)$

• particle density

$$n(0,\mu) = \int_{-\infty}^{\mu} \mathrm{d}\epsilon \, z(\epsilon)$$

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Zero temperature

if
$$T \rightarrow 0$$

 $\frac{1}{e^{(\epsilon - \mu)/k_{B}T} + 1} = \theta(\mu - \epsilon)$

• particle density

$$n(0,\mu) = \int_{-\infty}^{\mu} \mathrm{d}\epsilon \, z(\epsilon)$$

pressure

$$p(0,\mu) = rac{2}{3} \int_{-\infty}^{\mu} \mathrm{d}\epsilon \, z(\epsilon) \, \epsilon$$

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Zero temperature

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if
$$T \rightarrow 0$$

 $\frac{1}{e^{(\epsilon - \mu)/k_{B}T} + 1} = \theta(\mu - \epsilon)$

• particle density

$$n(0,\mu) = \int_{-\infty}^{\mu} \mathrm{d}\epsilon \, z(\epsilon)$$

pressure

$$p(0,\mu) = \frac{2}{3} \int_{-\infty}^{\mu} \mathrm{d}\epsilon \, z(\epsilon) \, \epsilon$$

• if particle density \bar{n} is given, then $\bar{n} = n(0, \epsilon_{\rm F}) = \int_{-\infty}^{\epsilon_{\rm F}} \mathrm{d}\epsilon \, z(\epsilon)$ defines Fermi energy

Atoms and so forth

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• Periodic system

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- Periodic system
- from small to huge molecules

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- Periodic system
- from small to huge molecules
- dielectrics, conductors, semiconductors, ferromagnets

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- Periodic system
- from small to huge molecules
- dielectrics, conductors, semiconductors, ferromagnets
- stability of normal matter

Atoms and so forth

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- Periodic system
- from small to huge molecules
- dielectrics, conductors, semiconductors, ferromagnets
- stability of normal matter
- stars can be stable even at T = 0

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Hydrodynamic equilibrium

• The mass within a sphere of radius R is $M(r) = 4\pi \int_0^r \mathrm{d}s \, s^2 \rho(s)$

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Hydrodynamic equilibrium

• The mass within a sphere of radius R is $M(r) = 4\pi \int_0^r \mathrm{d}s \, s^2 \rho(s)$

• gravitational force per unit volume

$$f(r) = -G \, \frac{\rho(r)M(r)}{r^2}$$

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Hydrodynamic equilibrium

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- The mass within a sphere of radius R is $M(r) = 4\pi \int_0^r \mathrm{d}s \, s^2 \rho(s)$
- gravitational force per unit volume

$$f(r) = -G \, \frac{\rho(r)M(r)}{r^2}$$

• pressure gradient and force must balance

$$p'(r)=f(r)$$

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Hydrodynamic equilibrium

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• The mass within a sphere of radius R is $M(r) = 4\pi \int_0^r \mathrm{d}s \, s^2 \rho(s)$

• gravitational force per unit volume

$$f(r) = -G \, \frac{\rho(r)M(r)}{r^2}$$

• pressure gradient and force must balance

p'(r) = f(r)

• relate pressure with mass density

Equation of state

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• electron gas at T = 0

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Equation of state

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• electron gas at
$$T = 0$$

• recall
 $z(\epsilon) = \frac{1}{\pi^2} \left\{ \frac{2m}{\hbar^2} \right\}^{3/2} \sqrt{\epsilon}$

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• electron gas at T = 0

recall

$$z(\epsilon) = rac{1}{\pi^2} \left\{ rac{2m}{\hbar^2}
ight\}^{3/2} \sqrt{\epsilon}$$

• particle density

$$n(0,\mu) = \frac{2}{3\pi^2} \left\{ \frac{2m}{\hbar^2} \right\}^{3/2} \mu^{3/2}$$

Equation of state

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Gibbs state

Fermions

Zero temperature

White dwarfs and neutron stars

Bosons

Black body radiation

Bose-Einstein condensation

Equation of state

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- electron gas at T = 0
- recall

$$z(\epsilon) = \frac{1}{\pi^2} \left\{ \frac{2m}{\hbar^2} \right\}^{3/2} \sqrt{\epsilon}$$

particle density

$$n(0,\mu) = \frac{2}{3\pi^2} \left\{ \frac{2m}{\hbar^2} \right\}^{3/2} \mu^{3/2}$$

pressure

$$p(0,\mu) = rac{4}{15\pi^2} \left\{rac{2m}{\hbar^2}
ight\}^{3/2} \mu^{5/2}$$

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Equation of state

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ight\}^{3/2} \mu^{5/2}$$

• eliminate chemical potential \hbar^2 = 10

$$p = a \frac{n}{m} n^{5/3}$$
 where $a = 1.2058$

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Equation of state

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pressure

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• eliminate chemical potential

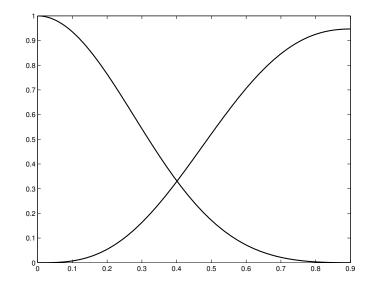
$$p = a \frac{\hbar^2}{m} n^{5/3}$$
 where $a = 1.2058$

• since one electron is related with two nucleons $p = a \frac{\hbar^2}{m_{\rm e}} \left\{ \frac{\rho}{2m_{\rm p}} \right\}^{5/3}$



Black bod radiation

Bose-Einstein condensation



Pressure p (decreasing) and mass M (increasing) of a white dwarf vs. distance r from the center in natural units. We have assumed zero temperature, two nucleons per electron which are treated non-relativistically. r = 1 corresponds to 6500 km, M = 1 to 0.85 sun masses.

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Crab nebula, the cloud of debris of the 1054 supernova

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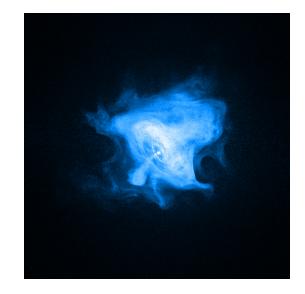
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There is a neutron star (pulsar) at its center.

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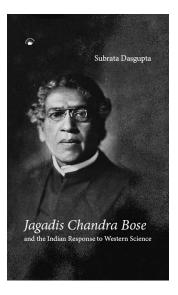
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Jagadris Chandra Bose, Indian physicist, 1858-1937

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• bosons have integer spin or integer helicity

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• bosons have integer spin or integer helicity

• such as the photon

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- bosons have integer spin or integer helicity
- such as the photon
- such as $(q\bar{q})$ states mesons

Bosons

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- bosons have integer spin or integer helicity
- such as the photon
- such as $(q\bar{q})$ states mesons
- or (ee) Cooper pairs

Bosons

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- bosons have integer spin or integer helicity
- such as the photon
- such as $(q\bar{q})$ states mesons
- or (ee) Cooper pairs
- or (ppnn), the helium nucleus or helium atom

Bosons

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- bosons have integer spin or integer helicity
- such as the photon
- such as $(q\bar{q})$ states mesons
- or (ee) Cooper pairs
- or (ppnn), the helium nucleus or helium atom
- or phonons, the quanta of lattice vibrations

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Many bosons

• recall the spin/statistics theorem



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- recall the spin/statistics theorem
- $[A_j, A_k] = [A_j^*, A_k^*] = 0$ commutators!

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- recall the spin/statistics theorem
- $[A_j, A_k] = [A_j^*, A_k^*] = 0$ commutators!
- $[A_j, A_k^*] = \delta_{jk} I$

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- recall the spin/statistics theorem
- $[A_j, A_k] = [A_j^*, A_k^*] = 0$ commutators!
- $[A_j, A_k^*] = \delta_{jk} I$
- $N_j = A_j^*, A_j$ number of particles in state j

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- $[A_j, A_k] = [A_j^*, A_k^*] = 0$ commutators!
- $[A_j, A_k^*] = \delta_{jk} I$
- $N_j = A_j^*, A_j$ number of particles in state j
- eigenvalues are $n = 0, 1, 2, \ldots$

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- recall the spin/statistics theorem
- $[A_j, A_k] = [A_j^*, A_k^*] = 0$ commutators!
- $[A_j, A_k^*] = \delta_{jk} I$
- $N_j = A_i^*, A_j$ number of particles in state j
- eigenvalues are $n = 0, 1, 2, \ldots$
- work out the trace, i. e. sum over eigenvalues n = 0, 1, 2, ...

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- recall the spin/statistics theorem
- $[A_j, A_k] = [A_j^*, A_k^*] = 0$ commutators!
- $[A_j, A_k^*] = \delta_{jk} I$
- $N_j = A_i^*, A_j$ number of particles in state j
- eigenvalues are $n = 0, 1, 2, \ldots$
- work out the trace, i. e. sum over eigenvalues n = 0, 1, 2, ...
- $N = \sum_{j} N_{j}$ total number of indistinguishable particles

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Free energy

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• energy is $H = \sum_j \epsilon_j A_j^* A_j + \dots$

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Free energy

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• energy is
$$H = \sum_{j} \epsilon_j A_j^* A_j + \dots$$

• free energy is

$$F = -k_{\rm B}T \ln {
m tr} ~ e^{\left(\mu N - H
ight)/k_{\rm B}T}$$

Free energy

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• energy is
$$H = \sum_j \epsilon_j A_j^* A_j + \dots$$

• free energy is

$$F = -k_{\rm B}T \ln {
m tr} \ e^{(\mu N - H)/k_{\rm B}T}$$

• N_i commute, therefore

$$e^{(\mu N - H)/k_{\rm B}T} = \prod_{j} e^{(\mu - \epsilon_j)N_j/k_{\rm B}T}$$

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Free energy

• energy is
$$H = \sum_j \epsilon_j A_j^* A_j + \dots$$

• free energy is

$$F = -k_{\rm B}T \ln {
m tr} \ {
m e}^{\left(\mu N - H\right)/k_{\rm B}T}$$

•
$$N_j$$
 commute, therefore
 $e^{(\mu N - H)/k_{\rm B}T} = \prod_j e^{(\mu - \epsilon_j)N_j/k_{\rm B}T}$

work out the trace, i. e. sum over eigenvalues
 n = 0, 1, 2, ...

tr
$$e^{(\mu - \epsilon_j)N_j/k_BT} = \sum_{n=0}^{\infty} \left\{ e^{(\mu - \epsilon_j)/k_BT} \right\}^n$$

= $\frac{1}{1 - e^{(\mu - \epsilon_j)/k_BT}}$

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Free energy ctd.

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• the free energy of a boson gas is $F = k_{\rm B}T \sum \ln \left(1 - e^{(\mu - \epsilon_j)/k_{\rm B}T}\right)$

$$= k_{\rm B}T \sum_{j} \ln \left(1 - e^{(\mu - \epsilon)/k_{\rm B}T}\right)$$
$$= k_{\rm B}T V \int d\epsilon z(\epsilon) \ln \left(1 - e^{(\mu - \epsilon)/k_{\rm B}T}\right)$$

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Free energy ctd.

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• the free energy of a boson gas is $F = k_{\rm B}T \sum_{j} \ln \left(1 - e^{(\mu - \epsilon_j)/k_{\rm B}T}\right)$ $= k_{\rm B}T V \left(\int de_{\rm C}(x) \ln \left(1 - e^{(\mu - \epsilon_j)/k_{\rm B}T}\right)\right)$

$$= k_{\rm B}T V \int d\epsilon z(\epsilon) \ln \left(1 - e^{(\mu - \epsilon)/\kappa_{\rm B}T}\right)$$

• compare with the free energy of a fermi gas

$${\cal F} = -{\sf k}_{\sf B} {\cal T} \; {\cal V} \int {
m d} \epsilon \, z(\epsilon) \, \ln \, \left(1 + \, e^{\left(\mu \, - \, \epsilon
ight) / {\sf k}_{\sf B} {\cal T}} \,
ight)$$

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Free energy ctd.

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- the free energy of a boson gas is $F = k_{\rm B}T \sum_{j} \ln \left(1 - e^{(\mu - \epsilon_j)/k_{\rm B}T}\right)$ $= k_{\rm B}T V \int d\epsilon z(\epsilon) \ln \left(1 - e^{(\mu - \epsilon)/k_{\rm B}T}\right)$
 - compare with the free energy of a fermi gas $F = -k_{\rm B}T V \int d\epsilon \, z(\epsilon) \ln \left(1 + e^{(\mu \epsilon)/k_{\rm B}T}\right)$
- recall spin/statistics theorem: again the +/- difference

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• photon energy is
$$\hbar\omega=
ho c$$

$$2\int \frac{\mathrm{d}^3 x \,\mathrm{d}^3 p}{(2\pi\hbar)^3} = V \int \frac{\mathrm{d}\omega \,\omega^2}{\pi^2 c^3}$$

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• photon energy is
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$$2\int \frac{\mathrm{d}^3 x \,\mathrm{d}^3 p}{(2\pi\hbar)^3} = V \int \frac{\mathrm{d}\omega \,\omega^2}{\pi^2 c^3}$$

$$\mu = 0$$

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Black body radiation

• photon energy is
$$\hbar\omega = pc$$

$$2\int \frac{\mathrm{d}^3 x \,\mathrm{d}^3 p}{(2\pi\hbar)^3} = V \int \frac{\mathrm{d}\omega \,\omega^2}{\pi^2 c^3}$$
$$\mu = 0$$

• black body free energy is

$$F = k_{\rm B}T V \int_0^\infty \frac{{\rm d}\omega \, \omega^2}{\pi^2 c^3} \ln \left(1 - e^{-\hbar\omega/k_{\rm B}T}\right)$$

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• photon energy is
$$\hbar\omega = pc$$

• density of states can be read off from

$$2\int \frac{\mathrm{d}^3 x \,\mathrm{d}^3 p}{(2\pi\hbar)^3} = V \int \frac{\mathrm{d}\omega \,\omega^2}{\pi^2 c^3}$$
$$\mu = 0$$

b black body free energy is

$$F = k_{\rm B}T V \int_0^\infty \frac{{\rm d}\omega \,\omega^2}{\pi^2 c^3} \ln \left(1 - e^{-\hbar\omega/k_{\rm B}T}\right)$$

• internal energy U

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• photon energy is
$$\hbar\omega=
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$$2\int \frac{\mathrm{d}^3 x \,\mathrm{d}^3 p}{(2\pi\hbar)^3} = V \int \frac{\mathrm{d}\omega \,\omega^2}{\pi^2 c^3}$$
$$\mu = 0$$

$$F = k_{\rm B}T V \int_0^\infty \frac{\mathrm{d}\omega \, \omega^2}{\pi^2 c^3} \ln \left(1 - e^{-\hbar\omega/k_{\rm B}T}\right)$$

- internal energy U
- $U = F + TS = F T\partial F / \partial T$

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• photon energy is
$$\hbar\omega = pc$$

$$2\int \frac{\mathrm{d}^3 x \,\mathrm{d}^3 p}{(2\pi\hbar)^3} = V \int \frac{\mathrm{d}\omega \,\omega^2}{\pi^2 c^3}$$
$$\mu = 0$$

$$F = k_{\rm B}T V \int_0^\infty \frac{\mathrm{d}\omega \, \omega^2}{\pi^2 c^3} \ln \left(1 - e^{-\hbar\omega/k_{\rm B}T}\right)$$

- internal energy U
- $U = F + TS = F T\partial F / \partial T$
- Planck's formula

$$U = V \int_0^\infty \frac{\mathrm{d}\omega \,\omega^2}{\pi^2 c^3} \, \frac{\hbar\omega}{e^{\hbar\omega/k_\mathrm{B}T} - \frac{1}{2}}$$

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Max Planck, German physicist, 1858-1947

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Particle density

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recall

$$\sum_{n=0}^{\infty} e^{n(\mu - \epsilon_j)/k_{\rm B}T} = \frac{1}{1 - e^{(\mu - \epsilon_j)/k_{\rm B}T}}$$

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Particle density

$$\sum_{n=0}^{\infty} e^{n(\mu - \epsilon_j)/k_{\rm B}T} = \frac{1}{1 - e^{(\mu - \epsilon_j)/k_{\rm B}T}}$$

• valid only if $\mu < \min \epsilon_j = 0$

Particle density

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• recall

$$\sum_{n=0}^{\infty} e^{n(\mu - \epsilon_j)/k_{\rm B}T} = \frac{1}{1 - e^{(\mu - \epsilon_j)/k_{\rm B}T}}$$

• valid only if $\mu < \min \epsilon_j = 0$

• $-\partial F/\partial \mu = \bar{N}$ is average particle number

Particle density

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• recall

$$\sum_{n=0}^{\infty} e^{n(\mu - \epsilon_j)/k_{\rm B}T} = \frac{1}{1 - e^{(\mu - \epsilon_j)/k_{\rm B}T}}$$

• valid only if $\mu < \min \epsilon_j = 0$

•
$$-\partial F/\partial \mu = \bar{N}$$
 is average particle number

• the average particle density \bar{n} is therefore $\bar{n}(T,\mu) = \int_0^\infty d\epsilon \, z(\epsilon) \, \frac{1}{e^{(\epsilon-\mu)/k_{\rm B}T} - 1}$

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• recall

$$\sum_{n=0}^{\infty} e^{n(\mu - \epsilon_j)/k_{\rm B}T} = \frac{1}{1 - e^{(\mu - \epsilon_j)/k_{\rm B}T}}$$

• valid only if $\mu < \min \epsilon_j = 0$

•
$$-\partial F/\partial \mu = \bar{N}$$
 is average particle number

• the average particle density \bar{n} is therefore $\bar{n}(T,\mu) = \int_0^\infty d\epsilon \, z(\epsilon) \, \frac{1}{e^{(\epsilon-\mu)/k_BT} - 1}$

• $\bar{n}(T,0)$ is maximal density:

$$\bar{n}_{\max}(T) = \int_0^\infty \mathrm{d}\epsilon \, z(\epsilon) \, \frac{1}{e^{\epsilon/\mathsf{k}_\mathsf{B}T} - 1}$$

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• If T decreases, so does $\bar{n}_{max}(T)$

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- If T decreases, so does $ar{n}_{ ext{max}}(T)$
- if \bar{n} is given, there is a temperature ${\cal T}_{\rm c}$ such that $\bar{n}_{\rm max}({\cal T}_{\rm c})=\bar{n}$

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- If T decreases, so does $ar{n}_{\max}(T)$
- if \bar{n} is given, there is a temperature ${\cal T}_{\rm c}$ such that $\bar{n}_{\rm max}({\cal T}_{\rm c})=\bar{n}$
- for even lower temperature, only a fraction of the particles are in thermal equilibrium

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- If ${\mathcal T}$ decreases, so does $ar{n}_{\max}({\mathcal T})$
- if \bar{n} is given, there is a temperature $T_{\rm c}$ such that $\bar{n}_{\rm max}(T_{\rm c})=\bar{n}$
- for even lower temperature, only a fraction of the particles are in thermal equilibrium

• the rest is in the multiply occupied ground state

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Bose-Einstein condensation

- If ${\mathcal T}$ decreases, so does $ar{n}_{\max}({\mathcal T})$
- if \bar{n} is given, there is a temperature $T_{\rm c}$ such that $\bar{n}_{\rm max}(T_{\rm c})=\bar{n}$
- for even lower temperature, only a fraction of the particles are in thermal equilibrium

- the rest is in the multiply occupied ground state
- supra-fluidity

Peter Hertel

Fundamenta particles

Spin and statistics

Many particles

Gibbs state

Fermions

Zero temperature

White dwarfs and neutron stars

Bosons

Black body radiation

Bose-Einstein condensation

Bose-Einstein condensation

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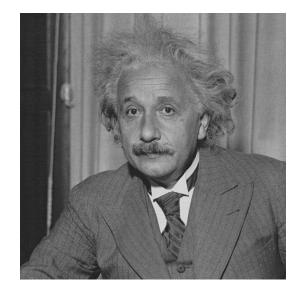
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Albert Einstein, German physicist, 1879-1955