Preventing waves from spreading

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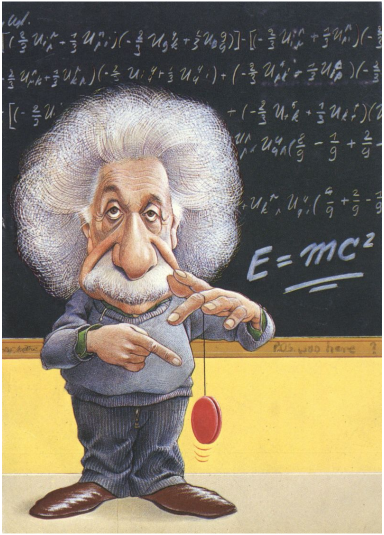

Make it as simple as possible, but not simpler.

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- light in free space: $\omega = c\,|\mathbf{k}|$

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- We normalize it to 1

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- group velocity

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- ... if the medium is homogeneous.

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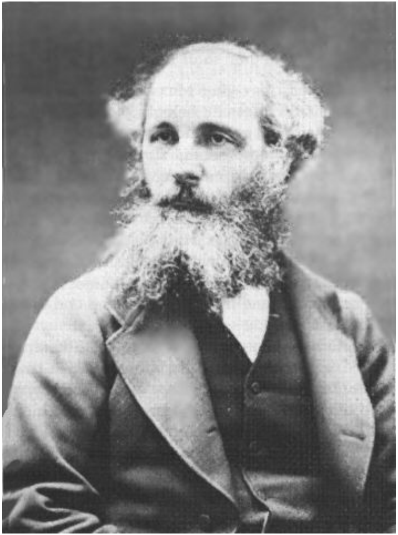

James Clerk Maxwell, 1831-1879

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- ϵ real and non-constant: dielectric waveguides

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