### Peter Hertel

Overview

Mode

Dielectric medium

Permittivity of metals

Electrical conductors

Faraday effect

# The Drude Model

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Paul Drude, German physicist, 1863-1906



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• The Drude model links optical and electric properties of a material with the behavior of its electrons or holes





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- Dielectric permittivity



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- Dielectric permittivity
- Permittivity of metals





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# • consider a typical electron



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- consider a typical electron
- denote by  $\boldsymbol{x} = \boldsymbol{x}(t)$  the deviation from its equilibrium position





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- consider a typical electron
- denote by  $\boldsymbol{x} = \boldsymbol{x}(t)$  the deviation from its equilibrium position
- external electric field strength  $\boldsymbol{E} = \boldsymbol{E}(t)$



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- external electric field strength  $\boldsymbol{E} = \boldsymbol{E}(t)$
- $m(\ddot{\boldsymbol{x}} + \Gamma \dot{\boldsymbol{x}} + \Omega^2 \boldsymbol{x}) = q\boldsymbol{E}$



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- electron mass m, charge q, friction coefficient  $m\Gamma,$  spring constant  $m\Omega^2$



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- Fourier transform this
- $m(-\omega^2 i\omega\Gamma + \Omega^2)\tilde{\boldsymbol{x}} = q\tilde{\boldsymbol{E}}$
- solution is

$$\tilde{\boldsymbol{x}}(\omega) = rac{q}{m} rac{\tilde{\boldsymbol{E}}(\omega)}{\Omega^2 - \omega^2 - \mathrm{i}\omega\Gamma}$$



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• dipole moment of typical electron is  $ilde{m{p}}=q ilde{m{x}}$ 



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- dipole moment of typical electron is  $ilde{m{p}}=q ilde{m{x}}$
- recall

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• dipole moment of typical electron is  $\tilde{p} = q\tilde{x}$ • recall

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 $\bullet\,$  there are N typical electrons per unit volume

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Faraday effect Hall effect • dipole moment of typical electron is  $\tilde{p} = q\tilde{x}$ • recall

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- polarization is 
$$ilde{m{P}}=Nq ilde{m{x}}=\epsilon_0\chi ilde{m{E}}$$

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Electrical conductor:

Faraday effect Hall effect • dipole moment of typical electron is  $\tilde{p} = q\tilde{x}$ • recall

$$\tilde{\boldsymbol{x}}(\omega) = \frac{q}{m} \frac{\tilde{\boldsymbol{E}}(\omega)}{\Omega^2 - \omega^2 - \mathrm{i}\omega\Gamma}$$

• there are  $\boldsymbol{N}$  typical electrons per unit volume

• polarization is 
$$ilde{m{P}}=Nq ilde{m{x}}=\epsilon_0\chi ilde{m{E}}$$

• susceptibility is

$$\chi(\omega) = \frac{Nq^2}{\epsilon_0 m} \frac{1}{\Omega^2 - \omega^2 - \mathrm{i}\omega\Gamma}$$

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$$\chi(0) = \frac{Nq^2}{\epsilon_0 m \Omega^2} > 0$$

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• ... as it should be

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• decompose susceptibility  $\chi(\omega) = \chi'(\omega) + i\chi''(\omega)$  into refractive part  $\chi'$  and absorptive part  $\chi''$ 

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- decompose susceptibility  $\chi(\omega) = \chi'(\omega) + i\chi''(\omega)$  into refractive part  $\chi'$  and absorptive part  $\chi''$
- Introduce  $R(\omega)=\chi(\omega)/\chi(0),\,s=\omega/\Omega$  and  $\gamma=\Gamma/\Omega$  as normalized quantities.

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- refraction

$$R'(s) = \frac{1 - s^2}{(1 - s^2)^2 + \gamma^2 s^2}$$

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$$R'(s) = \frac{1 - s^2}{(1 - s^2)^2 + \gamma^2 s^2}$$

• absorption

$$D''(s) = \frac{\gamma s}{(1 - s^2)^2 + \gamma^2 s^2}$$

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• absorption

$$D''(s) = \frac{\gamma s}{(1 - s^2)^2 + \gamma^2 s^2}$$

• limiting cases:  $s=0,\ s=1,\ s\to\infty,$  small  $\gamma$ 



Refractive part (blue) and absorptive part (red) of the susceptibility function  $\chi(\omega)$  scaled by the static value  $\chi(0)$ . The abscissa is  $\omega/\Omega$ .  $\Gamma/\Omega = 0.1$ 

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• For small frequencies (as compared with Ω) the susceptibility is practically real.


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- For small frequencies (as compared with  $\Omega$ ) the susceptibility is practically real.
- This is the realm of classical optics



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- For small frequencies (as compared with  $\Omega$ ) the susceptibility is practically real.
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- $\partial \chi / \partial \omega$  is positive normal dispersion



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- For small frequencies (as compared with  $\Omega$ ) the susceptibility is practically real.
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- $\partial\chi/\partial\omega$  is positive normal dispersion
- In the vicinity of  $\omega = \Omega$  absorption is large. Negative dispersion  $\partial \chi / \partial \omega$  is accompanied by strong absorption.

### **Discussion II**

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- In the vicinity of  $\omega = \Omega$  absorption is large. Negative dispersion  $\partial \chi / \partial \omega$  is accompanied by strong absorption.
- For very large frequencies again absorption is negligible, and the susceptibility is negative with normal dispersion. This applies to X rays.

### **Discussion II**

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- For very large frequencies again absorption is negligible, and the susceptibility is negative with normal dispersion. This applies to X rays.
- $\chi(\infty) = 0$  is required by first principles . . .

### Kramers-Kronig relation I

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# • $\chi(\omega)$ must be the Fourier transform of a causal response function $G = G(\tau)$

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- $\chi(\omega)$  must be the Fourier transform of a causal response function  $G=G(\tau)$
- as defined in

$$\boldsymbol{P}(t) = \epsilon_0 \int \mathrm{d}\tau G(\tau) \boldsymbol{E}(t-\tau)$$

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$$\boldsymbol{P}(t) = \epsilon_0 \int \mathrm{d}\tau G(\tau) \boldsymbol{E}(t-\tau)$$

• check this for

$$G(\tau) = a \int \frac{\mathrm{d}\omega}{2\pi} \frac{e^{-\mathrm{i}\omega\tau}}{\Omega^2 - \omega^2 - \mathrm{i}\omega\Gamma}$$

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poles at

$$\omega_{1,2} = -\frac{\mathrm{i}\Gamma}{2} \pm \bar{\omega}$$
 where  $\bar{\omega} = +\sqrt{\Omega^2 - \Gamma^2/4}$ 

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- Indeed,  $G(\tau)=0$  for  $\tau<0$ 

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• Indeed,  $G(\tau)=0$  for  $\tau<0$ 

• for 
$$\tau > 0$$
  

$$G(\tau) = \frac{Nq^2}{\epsilon_0 m} \frac{\sin \bar{\omega} \tau}{\bar{\omega}} e^{-\Gamma \tau/2}$$

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• causal response function:  $G(\tau) = \theta(\tau)G(\tau)$ 

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### • causal response function: $G(\tau) = \theta(\tau)G(\tau)$

• apply the convolution theorem

$$\chi(\omega) = \int \frac{\mathrm{d}u}{2\pi} \chi(u) \tilde{\theta}(\omega - u)$$

# Kramers-Kronig relation II

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• Fourier transform of Heaviside function is

$$\tilde{\theta}(\omega) = \lim_{0 < \eta \to 0} \frac{1}{\eta - i\omega}$$

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dispersion , or Kramers-Kronig relations

$$\chi'(\omega) = 2\Pr \int \frac{\mathrm{d}u}{\pi} \frac{u\chi''(u)}{u^2 - \omega^2}$$

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dispersion, or Kramers-Kronig relations

$$\chi'(\omega) = 2\Pr \int \frac{\mathrm{d}u}{\pi} \frac{u\chi''(u)}{u^2 - \omega^2}$$
$$\chi''(\omega) = 2\Pr \int \frac{\mathrm{d}u}{\pi} \frac{\omega\chi'(u)}{\omega^2 - u^2}$$

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### Dispersion of white light

# Free quasi-electrons

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### • consider a typical conduction band electron

# Free quasi-electrons

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- consider a typical conduction band electron
- it behaves as a free quasi-particle

### Free quasi-electrons

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• consider a typical conduction band electron

Free quasi-electrons

• it behaves as a free quasi-particle

• recall 
$$m(\ddot{\pmb{x}} + \Gamma\dot{\pmb{x}} + \Omega^2 \pmb{x}) = q\pmb{E}$$

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• consider a typical conduction band electron

Free quasi-electrons

- it behaves as a free quasi-particle
- recall  $m(\ddot{\pmb{x}} + \Gamma\dot{\pmb{x}} + \Omega^2 \pmb{x}) = q\pmb{E}$
- spring constant  $m\Omega^2$  vanishes

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· consider a typical conduction band electron

- it behaves as a free quasi-particle
- recall  $m(\ddot{\pmb{x}} + \Gamma\dot{\pmb{x}} + \Omega^2 \pmb{x}) = q\pmb{E}$
- spring constant  $m\Omega^2$  vanishes
- *m* is effective mass

### Peter Hertel

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Dielectric medium

Permittivity of metals

Electrical conductors

Faraday effect Hall effect

- · consider a typical conduction band electron
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$$\epsilon(\omega) = 1 - \frac{\omega_{\rm p}^2}{\omega^2 + \mathrm{i}\omega\Gamma}$$

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• plasma frequency  $\omega_{\mathrm{p}}$ 

$$\omega_{\rm p}^2 = \frac{Nq^2}{\epsilon_0 m}$$

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• plasma frequency  $\omega_{\mathrm{p}}$ 

$$\omega_{\rm p}^2 = \frac{Nq^2}{\epsilon_0 m}$$

- correction for  $\omega\gg\omega_{\rm p}$ 

$$\epsilon(\omega) = \epsilon_{\infty} - \frac{\omega_{\rm p}^2}{\omega^2 + i\omega\Gamma}$$

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### Example: gold

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• Drude model parameters for gold

### Example: gold



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• as determined by Johnson and Christy in 1972

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• 
$$\epsilon_{\infty} = 9.5$$



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- $\epsilon_{\infty} = 9.5$
- $\hbar\omega_{
  m p}=$  8.95 eV

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$$\hbar\omega_{
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- with these parameters the Drude model fits optical measurements well for  $\hbar\omega$  < 2.25 eV (green)
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- The refractive part of the permittivity can be large and negative while the absorptive part is small.

Example: gold

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- with these parameters the Drude model fits optical measurements well for  $\hbar\omega$  < 2.25 eV (green)
- The refractive part of the permittivity can be large and negative while the absorptive part is small.
- This allows surface plasmon polaritons (SPP)

Example: gold

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Refractive (blue) and absorptive part (red) of the permittivity function for gold. The abscissa is  $\hbar\omega$  in eV.

## Electrical conductivity

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### • consider a typical charged particle

## **Electrical conductivity**

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Faraday effect Hall effect • consider a typical charged particle

• recall  $m(\ddot{\pmb{x}} + \Gamma\dot{\pmb{x}} + \Omega^2 \pmb{x}) = q\pmb{E}$ 

# Electrical conductivity

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- recall  $m(\ddot{\pmb{x}} + \Gamma \dot{\pmb{x}} + \Omega^2 \pmb{x}) = q \pmb{E}$
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# Electrical conductivity

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# Electrical conductivity

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- recall

$$ilde{oldsymbol{x}}(\omega) = rac{q}{m} rac{ ilde{oldsymbol{E}}(\omega)}{\Omega^2 - \omega^2 - \mathrm{i}\omega\Gamma}$$

# **Electrical conductivity**

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• Ohm's law  $\tilde{J}(\omega) = \sigma(\omega)\tilde{E}(\omega)$ 

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• Ohm's law

$$\tilde{\boldsymbol{J}}(\omega) = \sigma(\omega)\tilde{\boldsymbol{E}}(\omega)$$

conductivity is

$$\sigma(\omega) = \frac{Nq^2}{m} \frac{-\mathrm{i}\omega}{\Omega^2 - \omega^2 - \mathrm{i}\omega\Gamma}$$

# Electrical conductivity

## **Electrical conductors**

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Faraday effect Hall effect • A material with  $\sigma(0) = 0$  is an electrical insulator. It cannot transport direct currents (DC).

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- A material with  $\sigma(0) = 0$  is an electrical insulator. It cannot transport direct currents (DC).
- A material with  $\sigma(0) > 0$  is an electrical conductor.

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- A material with  $\sigma(0) = 0$  is an electrical insulator. It cannot transport direct currents (DC).
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- which means

$$\sigma(\omega) = \frac{Nq^2}{m} \frac{1}{\Gamma - \mathrm{i}\omega}$$

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$$\sigma(\omega) = \frac{Nq^2}{m} \frac{1}{\Gamma - \mathrm{i}\omega}$$

or

$$\frac{\sigma(\omega)}{\sigma(0)} = \frac{1}{1 - i\omega/\Gamma}$$

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- Mode
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$$\sigma(\omega) = \frac{Nq^2}{m} \frac{1}{\Gamma - \mathrm{i}\omega}$$

- or
  - $\frac{\sigma(\omega)}{\sigma(0)} = \frac{1}{1 i\omega/\Gamma}$
- Note that the DC conductivity is always positive.

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Georg Simon Ohm, German physicist, 1789-1854

## External static magnetic field

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### • apply a quasi-static external induction ${\cal B}$

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## External static magnetic field

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- apply a quasi-static external induction  ${\cal B}$
- the typical electron obeys

$$m(\ddot{\boldsymbol{x}} + \Gamma \dot{\boldsymbol{x}} + \Omega^2 \boldsymbol{x}) = q(\boldsymbol{E} + \dot{\boldsymbol{x}} \times \boldsymbol{\mathcal{B}})$$

## External static magnetic field

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• Fourier transform this

$$m(-\omega^2 - i\omega\Gamma + \Omega^2)\tilde{\boldsymbol{x}} = q(\tilde{\boldsymbol{\mathcal{E}}} - i\omega\tilde{\boldsymbol{x}} \times \boldsymbol{\mathcal{B}})$$

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• assume 
$$\mathcal{B} = \mathcal{B} \hat{e}_z$$

## External static magnetic field

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Fourier transform this

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• assume 
$${\cal B}={\cal B}\hat{m e}_z$$

• assume circularly polarized light  

$$\tilde{\boldsymbol{\mathcal{E}}} = \tilde{E}_{\pm} \hat{\boldsymbol{e}}_{\pm}$$
 where  $\hat{\boldsymbol{e}}_{\pm} = (\hat{\boldsymbol{e}}_x + \mathrm{i}\hat{\boldsymbol{e}}_y)/\sqrt{2}$ 

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- try  $ilde{m{x}} = ilde{x}_\pm \hat{m{e}}_\pm$

• note 
$$\hat{m{e}}_{\pm} imes\hat{m{e}}_{z}=\mp\mathrm{i}\hat{m{e}}_{\pm}$$

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- apply a quasi-static external induction  ${\cal B}$
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$$m(-\omega^2 - i\omega\Gamma + \Omega^2)\tilde{\boldsymbol{x}} = q(\tilde{\boldsymbol{\mathcal{E}}} - i\omega\tilde{\boldsymbol{x}} \times \boldsymbol{\mathcal{B}})$$

- assume  $\mathcal{B} = \mathcal{B} \hat{e}_z$
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$$m(-\omega^2 - \mathrm{i}\omega\Gamma + \Omega^2)\tilde{x}_{\pm} = q(\tilde{E}_{\pm} \mp \omega \mathcal{B}\tilde{x}_{\pm})$$

## Faraday effect

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•  $m(-\omega^2 - i\omega\Gamma + \Omega^2)\tilde{x}_+ = q(\tilde{E}_+ \mp \omega \mathcal{B}\tilde{x}_+)$ 

## Faraday effect

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- $m(-\omega^2 i\omega\Gamma + \Omega^2)\tilde{x}_{\pm} = q(\tilde{E}_{\pm} \mp \omega \mathcal{B}\tilde{x}_{\pm})$
- therefore

$$\tilde{x}_{\pm} = \frac{q\tilde{E}_{\pm}}{m(\Omega^2 - i\omega\Gamma - \omega^2) \pm q\omega\mathcal{B}}$$

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recall  $\tilde{P} = Nq\tilde{x} = \epsilon_0\chi\tilde{E}$ 

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- effect of quasi-static induction  $\mathcal{B}$  is  $\chi_{\pm}(\omega) = \frac{Nq^2}{\epsilon_0 m} \frac{1}{\Omega^2 - i\omega\Gamma - \omega^2 \pm (q/m)\omega\mathcal{B}}$

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Michael Faraday, English physicist, 1791-1867
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• *B* is always small (in natural units)



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• *B* is always small (in natural units)

• 
$$\epsilon_{ij}(\omega; \mathcal{B}) = \epsilon_{ij}(\omega; 0) + iK(\omega)\epsilon_{ijk}\mathcal{B}_k$$

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- ${\mathcal B}$  is always small (in natural units)
- $\epsilon_{ij}(\omega; \mathcal{B}) = \epsilon_{ij}(\omega; 0) + iK(\omega)\epsilon_{ijk}\mathcal{B}_k$
- linear magneto-optic effect

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- $\epsilon_{ij}(\omega; \mathcal{B}) = \epsilon_{ij}(\omega; 0) + iK(\omega)\epsilon_{ijk}\mathcal{B}_k$
- linear magneto-optic effect
- Faraday constant is

$$K(\omega) = \frac{Nq^3}{\epsilon_0 m^2} \frac{\omega}{(\Omega^2 - i\omega\Gamma - \omega^2)^2}$$

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#### Peter Hertel

#### Overview

- Mode
- Dielectric medium
- Permittivity of metals
- Electrical conductors
- Faraday effect
- Hall effect

•  ${\mathcal B}$  is always small (in natural units)

Remarks

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- $\epsilon_{ij}(\omega; \mathcal{B}) = \epsilon_{ij}(\omega; 0) + iK(\omega)\epsilon_{ijk}\mathcal{B}_k$
- linear magneto-optic effect
- Faraday constant is

$$K(\omega) = \frac{Nq^3}{\epsilon_0 m^2} \frac{\omega}{(\Omega^2 - \mathrm{i}\omega\Gamma - \omega^2)^2}$$

• 
$$K(\omega)$$
 is real in transparency window

# Remarks

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Remarks

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$$K(\omega) = \frac{Nq^3}{\epsilon_0 m^2} \; \frac{\omega}{(\Omega^2 - \mathrm{i}\omega\Gamma - \omega^2)^2} \label{eq:K}$$

- $K(\omega)$  is real in transparency window
- i. e. if  $\omega$  is far away form  $\Omega$
- Faraday effect distinguishes between forward and backward propagation
- optical isolator

### Remarks

# Conduction in a magnetic field

Peter Hertel

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Hall effect

# Conduction in a magnetic field

• set the spring constant 
$$m\Omega^2=0$$

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# Conduction in a magnetic field

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- Mode
- Dielectric medium
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- Hall effect

- set the spring constant  $m\Omega^2=0$
- study AC electric field  $\tilde{\boldsymbol{\mathcal{E}}}$

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# Conduction in a magnetic field

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#### Peter Hertel

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- set the spring constant  $m\Omega^2=0$
- study AC electric field  $\tilde{\boldsymbol{\mathcal{E}}}$
- and static magnetic induction  ${oldsymbol{\mathcal{B}}}$
- solve

$$m(-\omega^2 - i\Gamma\omega)\tilde{\boldsymbol{x}} = q(\tilde{\boldsymbol{\mathcal{E}}} - i\omega\tilde{\boldsymbol{x}} \times \boldsymbol{\mathcal{B}})$$

# Conduction in a magnetic field

#### Peter Hertel

#### Overview

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• or

$$ilde{m{x}} = rac{q}{m} rac{1}{-\mathrm{i}\omega} rac{1}{\Gamma - \mathrm{i}\omega} \{ ilde{m{\mathcal{E}}} - \mathrm{i}\omega ilde{m{x}} imes m{\mathcal{B}} \}$$

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• by iteration

$$\tilde{\boldsymbol{x}} = \dots \{ \tilde{\boldsymbol{\mathcal{E}}} + rac{q}{m} rac{1}{\Gamma - \mathrm{i}\omega} \tilde{\boldsymbol{\mathcal{E}}} imes \boldsymbol{\mathcal{B}} \}$$

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- Ohmic current  $\propto \tilde{{\bm{\cal E}}}$  and Hall current  $\propto \tilde{{\bm{\cal E}}} \times {\bm{\cal B}}$ 

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Faraday effect

Hall effec



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### Hall effect, schematilly

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Hall effect



#### Peter Hertel

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### • Hall current usually forbidden by boundary conditions



Hall effect

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#### Overview

- Mode
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- Faraday effect
- Hall effect

• Hall current usually forbidden by boundary conditions

Hall effect

• Hall field

$$ilde{oldsymbol{\mathcal{E}}}_{\mathrm{H}} = -rac{q}{m}rac{1}{\Gamma-\mathrm{i}\omega} ilde{oldsymbol{\mathcal{E}}} imes oldsymbol{\mathcal{B}}$$

#### Peter Hertel

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• replace 
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 by  $ilde{m{\mathcal{E}}}+ ilde{m{\mathcal{E}}}_{
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•  $\tilde{\boldsymbol{\mathcal{E}}}_{\mathrm{H}} imes \boldsymbol{\mathcal{B}}$  can be neglected

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- current  $ilde{m{J}}(\omega)=\sigma(\omega) ilde{m{\mathcal{E}}}(\omega)$  as usual

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- ... if there is a dominant charge carrier.

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- ... if there is a dominant charge carrier.
- R has different sign for electrons and holes

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Edwin Hall, US-American physicist, 1855-1938