The Drude Model

Peter Hertel

University of Osnabrück, Germany

Lecture presented at APS, Nankai University, China

http://www.home.uni-osnabrueck.de/phertel

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Paul Drude, German physicist, 1863-1906

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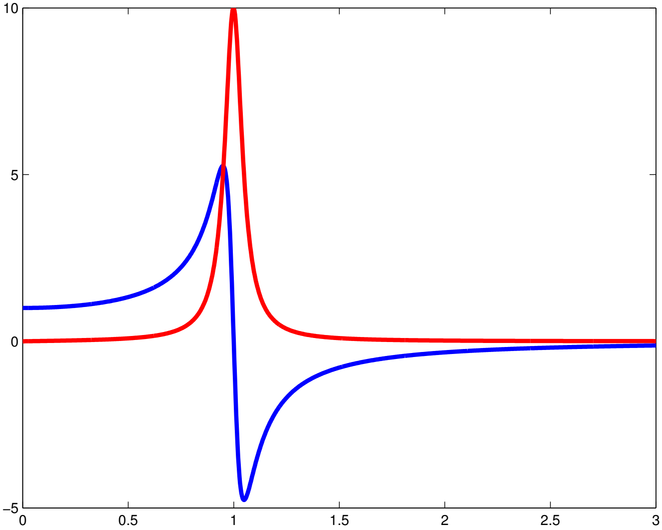
- limiting cases: $s = 0$, $s = 1$, $s \to \infty$, small γ



Refractive part (blue) and absorptive part (red) of the susceptibility function $\chi(\omega)$ scaled by the static value $\chi(0)$. The abscissa is ω/Ω. $\Gamma/\Omega = 0.1$

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- $\chi(\infty) = 0$ is required by first principles ...

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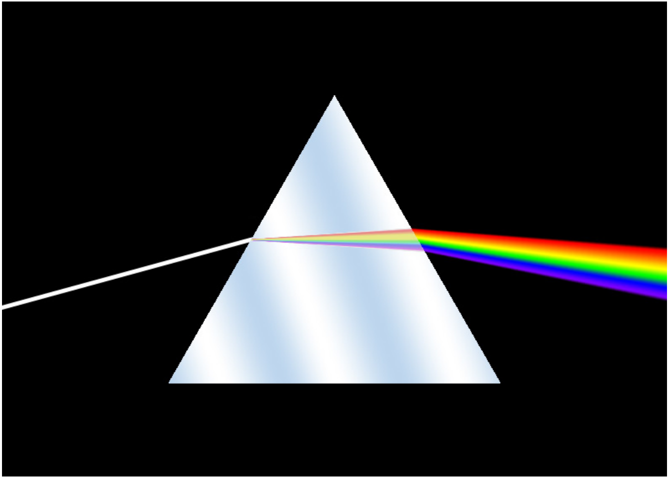

Dispersion of white light

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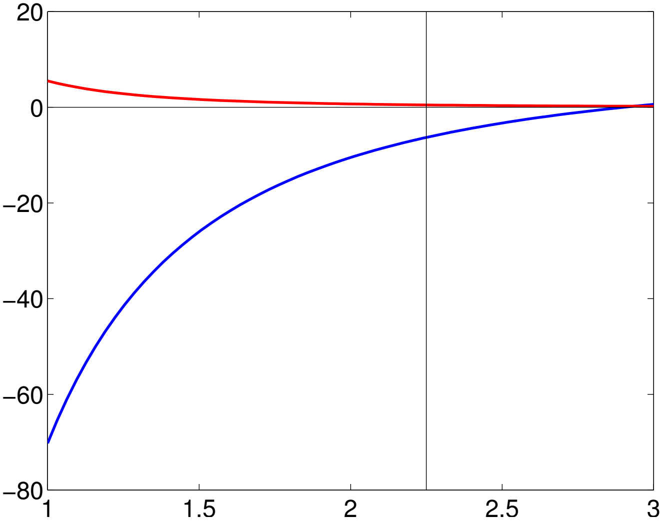
- This allows surface plasmon polaritons (SPP)



Refractive (blue) and absorptive part (red) of the permittivity function for gold. The abscissa is $\hbar\omega$ in eV.

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- Note that the DC conductivity is always positive.

Georg Simon Ohm, German physicist, 1789-1854

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- the typical electron obeys
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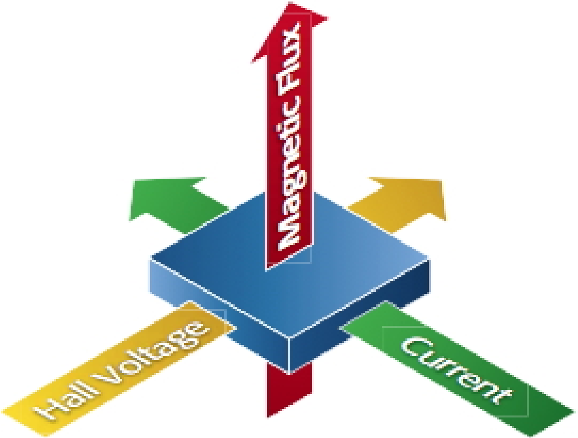

Hall effect, schematilly

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Edwin Hall, US-American physicist, 1855-1938