Metamaterials

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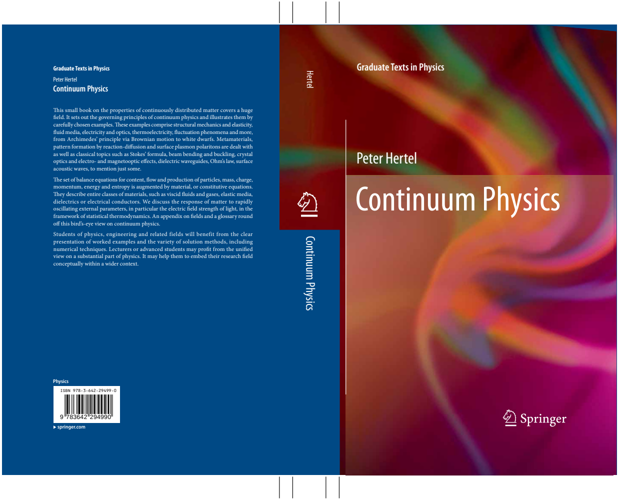
Spring 2012

Overview

- Metamaterials are produced artificially
- with strange optical properties
- for instance negative dielectric permittivity
- *and* negative magnetic permeability
- split ring resonator SRR
- Snellius law and more
- Same for a backward material
- Applications


Graduate Texts in Physics
Peter Hertel
Continuum Physics
This small book on the properties of continuously distributed matter covers a huge field. It sets out the governing principles of continuum physics and illustrates them by carefully chosen examples. These examples comprise structural mechanics and elasticity, fluid media, electricity and optics, thermoelectricity, fluctuation phenomena and more, from Archimedes' principle via Brownian motion to white dwarfs. Metamaterials, pattern formation by reaction-diffusion and surface plasmon polaritons are dealt with as well as classical topics such as Stokes' formula, beam bending and buckling, crystal optics and electro- and magnetooptic effects, dielectric waveguides, Ohm's law, surface acoustic waves, to mention just some.
The set of balance equations for content, flow and production of particles, mass, charge, momentum, energy and entropy is augmented by material, or constitutive equations. They describe entire classes of materials, such as viscid fluids and gases, elastic media, dielectrics or electrical conductors. We discuss the response of matter to rapidly oscillating external parameters, in particular the electric field strength of light, in the framework of statistical thermodynamics. An appendix on fields and a glossary round off this bird's-eye view on continuum physics.
Students of physics, engineering and related fields will benefit from the clear presentation of worked examples and the variety of solution methods, including numerical techniques. Lecturers or advanced students may profit from the unified view on a substantial part of physics. It may help them to embed their research field conceptually within a wider context.
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ISBN 978-3-642-29499-0
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Drude model

- harmonically oscillating field

$$\boldsymbol{E}(t) = \tilde{\boldsymbol{E}}\,\mathrm{e}^{-\mathrm{i}\omega t}$$

- elastically bound damped electron

$$m\{\ddot{\boldsymbol{x}} + \Gamma\dot{\boldsymbol{x}} + \Omega^2\boldsymbol{x}\} = q\boldsymbol{E}$$

- Fourier transform it

$$\tilde{\boldsymbol{x}} = \frac{q}{m}\frac{\tilde{\boldsymbol{E}}}{-\omega^2 - \mathrm{i}\Gamma\omega + \Omega^2}$$

- polarization for N electrons per unit volume

$$\tilde{\boldsymbol{P}} = \frac{Nq^2}{m}\frac{\tilde{\boldsymbol{E}}}{-\omega^2 - \mathrm{i}\Gamma\omega + \Omega^2}$$

- magnetization$\tilde{\boldsymbol{M}}$ negligibly small

Dielectric permittivity

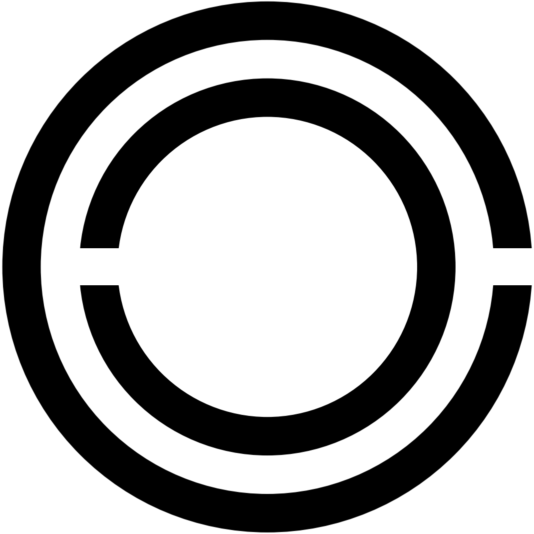
- susceptibility χ defined by
  $\tilde{\boldsymbol{P}} = \chi(\omega)\epsilon_0 \tilde{\boldsymbol{E}}$
- generalized Drude model

$$\chi(\omega) = \sum_a \frac{f_a \Omega_a^2}{-\omega^2 - \mathrm{i}\Gamma_a \omega + \Omega_a^2}$$

- sum over all resonance frequencies Ω_a with weights f_a (oscillator strength)
- a variant of this is also known as Sellmeier's formula
- permittivity function $\epsilon(\omega)$ defined by
  $\tilde{\boldsymbol{D}} = \epsilon(\omega)\epsilon_0 \tilde{\boldsymbol{E}}$
- $\epsilon(\omega) = 1 + \chi(\omega)$
- permittivity of natural materials, like solids or liquids

Artificial resonators

- If you want strange optical effects ...
- you must provide for strange resonators,
- not just elastically bound damped electrons.
- Metamaterials are arrays of resonating circuits
- the dimension of which is small if compared with the wavelength
- can be realized easily for microwaves
- Optical metamaterials require advanced nano-technology
- such as self-assembling
- as of today: science fiction



The split ring resonator. Inductance L and capacitance C. Resonance frequency is $\Omega = 1/\sqrt{LC}$.

Maxwell's equations

- Maxwell's equations for plain waves
  $f(t,\boldsymbol{x}) = \tilde{f}\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}$
- curl of the magnetic field
  $\boldsymbol{k} \times \tilde{\boldsymbol{H}} = -\omega\epsilon(\omega)\epsilon_0\tilde{\boldsymbol{E}}$
- curl of the electric field
  $\boldsymbol{k} \times \tilde{\boldsymbol{E}} = \omega\mu(\omega)\mu_0\tilde{\boldsymbol{H}}$
- $\boldsymbol{k} = k\hat{\boldsymbol{k}}$ and $\tilde{\boldsymbol{E}} = \tilde{E}\hat{\boldsymbol{e}}$
- wave number, propagation direction, amplitude, polarization vector
- solution
  $\tilde{\boldsymbol{H}} = \frac{k\tilde{E}}{\omega\mu\mu_0}\hat{\boldsymbol{k}} \times \hat{\boldsymbol{e}}$

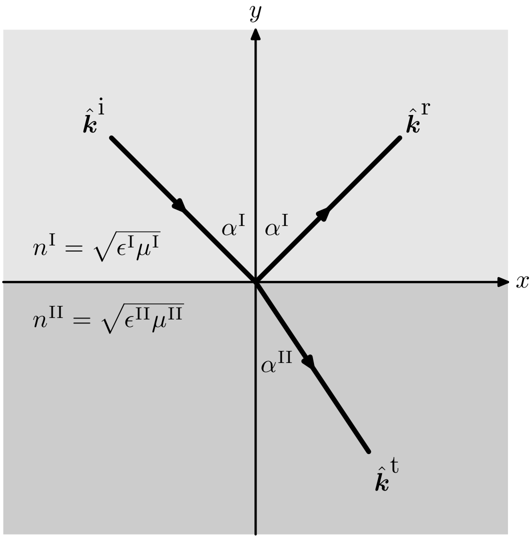
Maxwell's equations ctd.

- dispersion relation

  $k^2 c^2 = \omega^2\, \epsilon(\omega)\mu(\omega)$
- or, with $k_0 = \omega/c$

  $k = n(\omega)\, k_0$ where $n(\omega) = +\sqrt{\epsilon(\omega)\mu(\omega)}$
- Poynting vector $\boldsymbol{S} = 2$ Re $\boldsymbol{E}^* \times \boldsymbol{H}$

  $\boldsymbol{S} = \dfrac{2n}{c\mu\mu_0}|\tilde{E}|^2\hat{\boldsymbol{k}}$
- n real if $\epsilon > 0$ and $\mu > 0$
- this is a forward medium
- n also real if $\epsilon < 0$ and $\mu < 0$
- backward medium
- *meta instead of natural material*



Incident, transmitted and reflected beam at interface of normal materials

Continuity requirements

- let us study perpendicularly polarized light
- only $E = E_z$ components do not vanish
- $y > 0$: incident+reflected; $y < 0$: transmitted

$$E^{\mathrm{i}} = E^{\mathrm{i}}\,\mathrm{e}^{\mathrm{i}k_0 n^{\mathrm{I}}(\sin\alpha^{\mathrm{i}} x - \cos\alpha^{\mathrm{i}} y)}$$

$$E^{\mathrm{r}} = E^{\mathrm{r}}\,\mathrm{e}^{\mathrm{i}k_0 n^{\mathrm{I}}(\sin\alpha^{\mathrm{r}} x + \cos\alpha^{\mathrm{r}} y)}$$

$$E^{\mathrm{t}} = E^{\mathrm{t}}\,\mathrm{e}^{\mathrm{i}k_0 n^{\mathrm{II}}(\sin\alpha^{\mathrm{t}} x - \cos\alpha^{\mathrm{t}} y)}$$

- tangential components must be continuous
- $E^{\mathrm{i}} + E^{\mathrm{r}} = E^{\mathrm{t}}$
- incident and reflected beam angles are equal, Snell's law

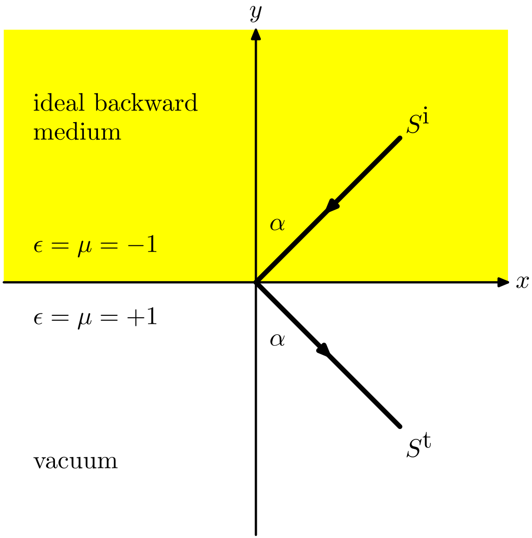
$$n^{\mathrm{I}}\sin\alpha^{\mathrm{i}} = n^{\mathrm{I}}\sin\alpha^{\mathrm{r}} = n^{\mathrm{II}}\sin\alpha^{\mathrm{t}}$$

- H_x, H_z and μH_y must be continuous
- satisfied if

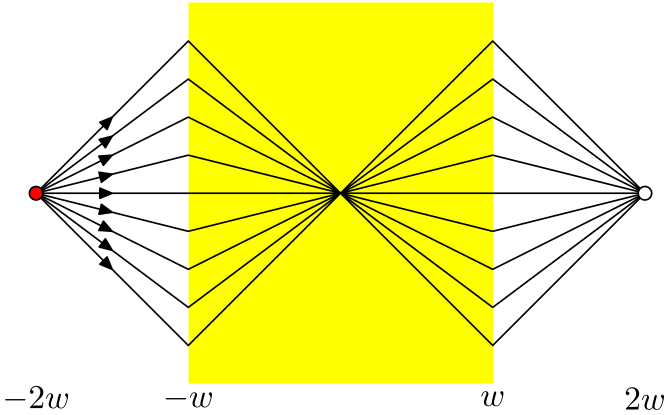
$$\frac{n^{\mathrm{I}}\cos\alpha^{\mathrm{I}}}{\mu^{\mathrm{I}}}(E^{\mathrm{i}} - E^{\mathrm{r}}) = \frac{n^{\mathrm{II}}\cos\alpha^{\mathrm{II}}}{\mu^{\mathrm{II}}}E^{\mathrm{t}}$$

Refraction in a backward medium

- we discuss an ideal backward medium
- $\epsilon = -1$ and $\mu = -1$
- and its plane interface with vacuum
- refractive indexes of both media are $n^{\rm I} = n^{\rm II} = 1$
- all angles are equal
- as before: $E^{\rm r} + E^{\rm i} = E^{\rm t}$
- different: $E^{\rm r} - E^{\rm i} = E^{\rm t}$
- therefore $E^{\rm i} = 0$ and $E^{\rm r} = E^{\rm t}$
- the formerly reflected beam is now incident
- the formerly incident beam vanishes



Power flow of a a beam which passes from an ideal backward medium to vacuum.



A parallel slab of ideal backward material. It maps the upward object at the left into an equally large upward image at the right. No diffraction limitation!

Summary

- Metamaterials consist of regularly arranged LC resonators
- Realized for microwaves, still science fiction for optics
- progress in nano technology expected
- in particular, self-assembling
- metamaterials show positive and negative permittivities and permeabilities
- if both are negative - in a certain frequency range - the energy flows counter to wave propagation
- metamaterials may be backward
- strange behavior at interfaces between normal and metamaterial
- example: a new microscope not restricted by the diffraction limit