Dielectric Susceptibility

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- $M(\tau) = U_{-\tau} A U_\tau$ where $U_\tau = e^{-\frac{i}{\hbar}\tau H}$

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$$H = \sum_a \frac{\mathbf{p}_a^2}{2m_a} + \frac{1}{4\pi\epsilon_0} \sum_{b>a} \frac{q_b q_a}{|\mathbf{x}_b - \mathbf{x}_a|}$$

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- Matrix of influence functions

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- where the susceptibility tensor is

$$\chi_{ij}(\omega, \mathbf{q}) = \frac{1}{\epsilon_0} \int_0^\infty d\tau \; e^{i\omega t} \int d^3\xi \; e^{-i\mathbf{q} \cdot \boldsymbol{\xi}} \; \Gamma_{ij}(\tau, \boldsymbol{\xi})$$