Natural units

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Overview

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- Normal matter

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- Atomic units

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- SI to AU conversion

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- the powers of SI units

	m	kg	s	A
$\hbar$	2	1	-1	0
e	0	0	1	1
m	0	1	0	0
$4\pi\epsilon_0$	-3	-1	4	2

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- read: s = number multiplied by $\hbar^3 e^{-4} m^{-1} (4\pi\epsilon_0)^2$ etc.
- find out number and exponents for arbitrary SI unit

```
function [value,power]=atomic_unit(si)
val=[1.05457e-34,1.60218e-19,9.10938e-31,4*pi*8.85419
dim=[2 1 -1 0; 0 0 1 1; 0 1 0 0; -3 -1 4 2];
mid=round(inv(dim));
power=si*mid;
value=prod(val.^power);
```

```
>> length = [0 1 0 0];
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>> [astar,apow]=atomic_unit(length);
>> astar
   5.2917e-11
>> apow
   2    -2    -1     1
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- i. e. Bohr's radius

$$a_* = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 5.2917 \times 10^{-11}\ \mathrm{m}$$

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$$\langle R\rangle = \frac{\int_0^\infty \mathrm{d}r\, r^2\, e^{-r}\, r\, e^{-r}}{\int_0^\infty \mathrm{d}r\, r^2\, e^{-r}\, 1\, e^{-r}} = 1$$

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- $\hbar\omega = 0.072\, E_* = 1.96$ eV

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- lithium niobate is rather resilient to electrical fields

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- steel : $E = 200 \text{ GPa}$

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- $N \approx 1$ and $\Omega \approx 1$ in natural units!