Crystal optics

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- Drude model

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- $\epsilon_{ij}(\omega) = \delta_{ij} + \chi_{ij}(\omega)$ where $\chi_{ij} = \tilde{G}_{ij}$

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- i. e. the conversion of field energy into internal energy

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- to be solved is the mode equation
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- extraordinary beam is polarized $\parallel$ optical axis
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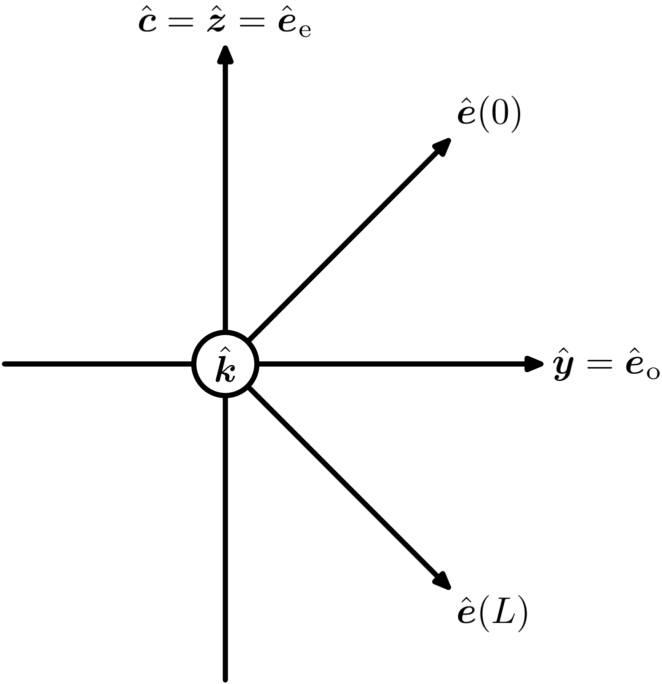
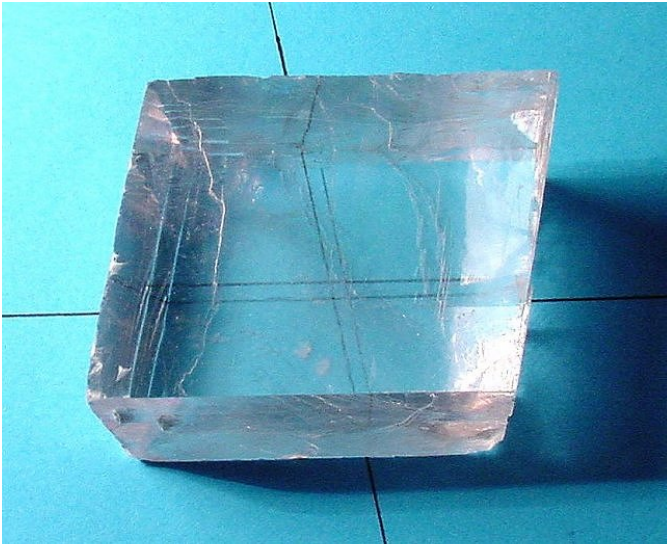
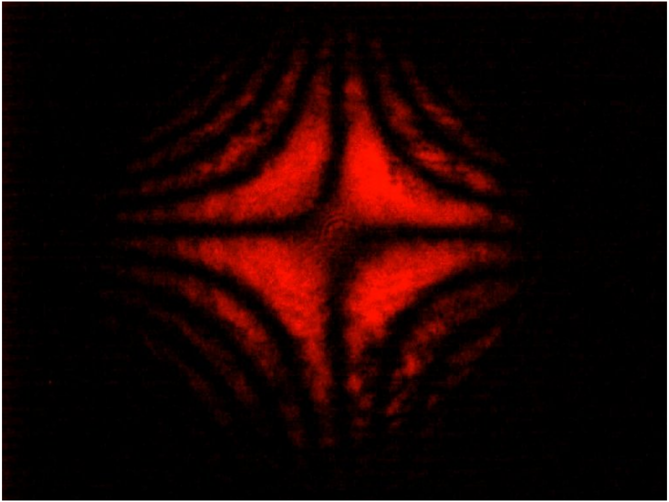

$\hat{\boldsymbol{c}} = \hat{\boldsymbol{z}} = \hat{\boldsymbol{e}}_{\mathrm{e}}$
$\hat{\boldsymbol{e}}(0)$
$\hat{\boldsymbol{k}}$
$\hat{\boldsymbol{y}} = \hat{\boldsymbol{e}}_{\mathrm{o}}$
$\hat{\boldsymbol{e}}(L)$

A 45 degree polarized wave enters the crystal and leaves it at -45 degrees polarization.



Birefringence, or double refraction, by calcite



Normally isotropic polymers become birefringent when stressed. Observed with a polarizer.

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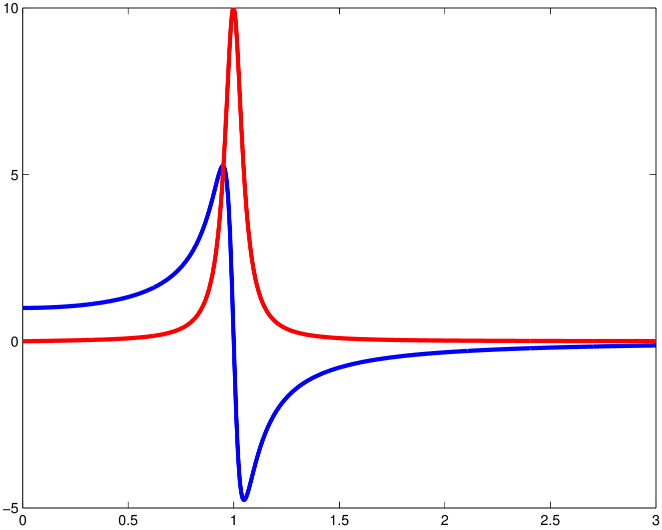

Real (blue) and imaginary part (red) of susceptibility $\chi(\omega)$ relative to $\chi(0)$ over ω/Ω. $\Gamma/\Omega = 0.1$