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- this is a Green's or influence function

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- Suszeptibility tensor defined by

$$\chi_{ij}(\omega, \mathbf{q}) = \frac{1}{\epsilon_0} \int_0^\infty d\tau \; e^{\, i\omega\tau} \int d^3\xi \; e^{\, -i\mathbf{q} \cdot \boldsymbol{\xi}} \; \Gamma_{ij}(\tau, \boldsymbol{\xi})$$

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- spatial dispersion, optical activity