Onsager relations

Peter Hertel

University of Osnabrück, Germany

April 6, 2010

Symmetry of static susceptibilities

- $H(\lambda) = H - \sum_r \lambda_r V_r$

Symmetry of static susceptibilities

- $H(\lambda) = H - \sum_r \lambda_r V_r$
- $F(\beta, \lambda) = -(1/\beta) \ln \operatorname{tr} e^{-\beta H(\lambda)}$

Symmetry of static susceptibilities

- $H(\lambda) = H - \sum_r \lambda_r V_r$
- $F(\beta, \lambda) = -(1/\beta) \ln \text{tr } e^{-\beta H(\lambda)}$
- $< V_r >_\lambda = -\partial F(\beta, \lambda)/\partial \lambda_r = \ldots$

Symmetry of static susceptibilities

- $H(\lambda) = H - \sum_r \lambda_r V_r$
- $F(\beta, \lambda) = -(1/\beta) \ln \text{ tr } e^{-\beta H(\lambda)}$
- $< V_r >_\lambda = -\partial F(\beta, \lambda) / \partial \lambda_r = \ldots$
- $\ldots = < V_r > + \sum_s \lambda_s \, \partial < V_r >_\lambda / \partial \lambda_s |_{\lambda = 0}$

Symmetry of static susceptibilities

- $H(\lambda) = H - \sum_r \lambda_r V_r$
- $F(\beta, \lambda) = -(1/\beta) \ln \text{tr}\, e^{-\beta H(\lambda)}$
- $< V_r >_\lambda = -\partial F(\beta, \lambda)/\partial \lambda_r = \ldots$
- $\ldots = < V_r > + \sum_s \lambda_s \, \partial < V_r >_\lambda / \partial \lambda_s |_{\lambda=0}$
- This says

$$\chi_{rs}(0) = \int_0^\infty d\tau \, \Gamma_{rs}(\tau) = - \left. \frac{\partial^2 F(\beta, \lambda)}{\partial \lambda_r \partial \lambda_s} \right|_{\lambda=0}$$

Symmetry of static susceptibilities

- $H(\lambda) = H - \sum_r \lambda_r V_r$
- $F(\beta, \lambda) = -(1/\beta) \ln \text{tr}\, e^{-\beta H(\lambda)}$
- $< V_r >_\lambda = -\partial F(\beta, \lambda)/\partial \lambda_r = \ldots$
- $\ldots = < V_r > + \sum_s \lambda_s \, \partial < V_r >_\lambda / \partial \lambda_s |_{\lambda=0}$
- This says

$$\chi_{rs}(0) = \int_0^\infty d\tau \, \Gamma_{rs}(\tau) = - \left. \frac{\partial^2 F(\beta, \lambda)}{\partial \lambda_r \partial \lambda_s} \right|_{\lambda=0}$$

- Static susceptibilities are symmetric

Time reversal

- Time reversed operator $A^\star$

Time reversal

- Time reversed operator $A^\star$
- $(A\phi)^* = A^\star\phi^*$ where $\phi = \phi(x_1, x_2, \dots)$

Time reversal

- Time reversed operator $A^\star$
- $(A\phi)^* = A^\star \phi^*$ where $\phi = \phi(x_1, x_2, \dots)$
- $A + B \longrightarrow A^\star + B^\star$, but $zA \longrightarrow z^* A^\star$

Time reversal

- Time reversed operator $A^\star$
- $(A\phi)^* = A^\star \phi^*$ where $\phi = \phi(x_1, x_2, \dots)$
- $A + B \longrightarrow A^\star + B^\star$, but $zA \longrightarrow z^* A^\star$
- $\mathbf{x}^\star = \mathbf{x}$ and $\mathbf{p}^\star = -\mathbf{p}$

Time reversal

- Time reversed operator $A^\star$
- $(A\phi)^* = A^\star \phi^*$ where $\phi = \phi(x_1, x_2, \dots)$
- $A + B \longrightarrow A^\star + B^\star$, but $zA \longrightarrow z^* A^\star$
- $\mathbf{x}^\star = \mathbf{x}$ and $\mathbf{p}^\star = -\mathbf{p}$
- motion inversion

Time reversal

- Time reversed operator $A^\star$
- $(A\phi)^* = A^\star \phi^*$ where $\phi = \phi(x_1, x_2, \dots)$
- $A + B \to A^\star + B^\star$, but $zA \to z^* A^\star$
- $\mathbf{x}^\star = \mathbf{x}$ and $\mathbf{p}^\star = -\mathbf{p}$
- motion inversion
- $A\phi = \lambda\phi$ implies $A^\star \phi^* = \lambda^* \phi^*$

Time reversal

- Time reversed operator $A^\star$
- $(A\phi)^* = A^\star \phi^*$ where $\phi = \phi(x_1, x_2, \dots)$
- $A + B \to A^\star + B^\star$, but $zA \to z^* A^\star$
- $\mathbf{x}^\star = \mathbf{x}$ and $\mathbf{p}^\star = -\mathbf{p}$
- motion inversion
- $A\phi = \lambda\phi$ implies $A^\star \phi^* = \lambda^* \phi^*$
- time reversed observable is an observable

Time reversal

- Time reversed operator $A^\star$
- $(A\phi)^* = A^\star\phi^*$ where $\phi = \phi(x_1, x_2, \dots)$
- $A + B \to A^\star + B^\star$, but $zA \to z^* A^\star$
- $\mathbf{x}^\star = \mathbf{x}$ and $\mathbf{p}^\star = -\mathbf{p}$
- motion inversion
- $A\phi = \lambda\phi$ implies $A^\star\phi^* = \lambda^*\phi^*$
- time reversed observable is an observable
- time reversed state is a state

Time reversal

- Time reversed operator $A^{\star}$
- $(A\phi)^{*} = A^{\star}\phi^{*}$ where $\phi = \phi(x_1, x_2, \dots)$
- $A + B \rightarrow A^{\star} + B^{\star}$, but $zA \rightarrow z^{*}A^{\star}$
- $\mathbf{x}^{\star} = \mathbf{x}$ and $\mathbf{p}^{\star} = -\mathbf{p}$
- motion inversion
- $A\phi = \lambda\phi$ implies $A^{\star}\phi^{*} = \lambda^{*}\phi^{*}$
- time reversed observable is an observable
- time reversed state is a state
- $\mathrm{tr}\, WM = \mathrm{tr}\, W^{\star}M^{\star}$

Time reversal

- Time reversed operator $A^\star$
- $(A\phi)^* = A^\star \phi^*$ where $\phi = \phi(x_1, x_2, \ldots)$
- $A + B \longrightarrow A^\star + B^\star$, but $zA \longrightarrow z^* A^\star$
- $\mathbf{x}^\star = \mathbf{x}$ and $\mathbf{p}^\star = -\mathbf{p}$
- motion inversion
- $A\phi = \lambda\phi$ implies $A^\star \phi^* = \lambda^* \phi^*$
- time reversed observable is an observable
- time reversed state is a state
- $\operatorname{tr} WM = \operatorname{tr} W^\star M^\star$
- parity $M^\star = \eta M$, $\eta = \pm 1$

Time reversal

- Time reversed operator $A^\star$
- $(A\phi)^* = A^\star \phi^*$ where $\phi = \phi(x_1, x_2, \dots)$
- $A + B \to A^\star + B^\star$, but $zA \to z^* A^\star$
- $\mathbf{x}^\star = \mathbf{x}$ and $\mathbf{p}^\star = -\mathbf{p}$
- motion inversion
- $A\phi = \lambda\phi$ implies $A^\star \phi^* = \lambda^* \phi^*$
- time reversed observable is an observable
- time reversed state is a state
- $\operatorname{tr} WM = \operatorname{tr} W^\star M^\star$
- parity $M^\star = \eta M$, $\eta = \pm 1$
- position/momentum operators have even/odd parity

Time reversal

- Time reversed operator $A^\star$
- $(A\phi)^* = A^\star \phi^*$ where $\phi = \phi(x_1, x_2, \dots)$
- $A + B \to A^\star + B^\star$, but $zA \to z^* A^\star$
- $\mathbf{x}^\star = \mathbf{x}$ and $\mathbf{p}^\star = -\mathbf{p}$
- motion inversion
- $A\phi = \lambda\phi$ implies $A^\star \phi^* = \lambda^* \phi^*$
- time reversed observable is an observable
- time reversed state is a state
- $\text{tr}\, WM = \text{tr}\, W^\star M^\star$
- parity $M^\star = \eta M$, $\eta = \pm 1$
- position/momentum operators have even/odd parity
- angular momenta have odd parity, magnetic moments as well

Time reversal (ctd.)

- Static magnetic induction $\bar{B}$

Time reversal (ctd.)

- Static magnetic induction $\bar{B}$
- $H(\bar{B}) = H(0) - \sum_i \mu_i \bar{B}_i$

Time reversal (ctd.)

- Static magnetic induction $\bar{B}$
- $H(\bar{B}) = H(0) - \sum_i \mu_i \bar{B}_i$
- $H(\bar{B})^\star = H(-\bar{B})$ for the Hamiltonian

Time reversal (ctd.)

- Static magnetic induction $\bar{B}$
- $H(\bar{B}) = H(0) - \sum_i \mu_i \bar{B}_i$
- $H(\bar{B})^\star = H(-\bar{B})$ for the Hamiltonian
- $G(\bar{B})^\star = G(-\bar{B})$ for the Gibbs state

Time reversal (ctd.)

- Static magnetic induction $\bar{B}$
- $H(\bar{B}) = H(0) - \sum_i \mu_i \bar{B}_i$
- $H(\bar{B})^\star = H(-\bar{B})$ for the Hamiltonian
- $G(\bar{B})^\star = G(-\bar{B})$ for the Gibbs state
- $U_t(\bar{H})^\star = U_{-t}(-\bar{B})$

Time reversal (ctd.)

- Static magnetic induction $\bar{B}$
- $H(\bar{B}) = H(0) - \sum_i \mu_i \bar{B}_i$
- $H(\bar{B})^\star = H(-\bar{B})$ for the Hamiltonian
- $G(\bar{B})^\star = G(-\bar{B})$ for the Gibbs state
- $U_t(\bar{H})^\star = U_{-t}(-\bar{B})$
- therefore time reversal, however $\bar{B} \to -\bar{B}$

Time reversal (ctd.)

- Static magnetic induction $\bar{B}$
- $H(\bar{B}) = H(0) - \sum_i \mu_i \bar{B}_i$
- $H(\bar{B})^\star = H(-\bar{B})$ for the Hamiltonian
- $G(\bar{B})^\star = G(-\bar{B})$ for the Gibbs state
- $U_t(\bar{H})^\star = U_{-t}(-\bar{B})$
- therefore time reversal, however $\bar{B} \to -\bar{B}$
- Maxwell's equations

Time reversal (ctd.)

- Static magnetic induction $\bar{B}$
- $H(\bar{B}) = H(0) - \sum_i \mu_i \bar{B}_i$
- $H(\bar{B})^\star = H(-\bar{B})$ for the Hamiltonian
- $G(\bar{B})^\star = G(-\bar{B})$ for the Gibbs state
- $U_t(\bar{H})^\star = U_{-t}(-\bar{B})$
- therefore time reversal, however $\bar{B} \to -\bar{B}$
- Maxwell's equations
- $\epsilon_0 \boldsymbol{\nabla} \cdot \mathbf{E} = \rho$ and $(1/\mu_0) \boldsymbol{\nabla} \times \mathbf{B} - \epsilon_0 \dot{\mathbf{E}} = \mathbf{j}$

Time reversal (ctd.)

- Static magnetic induction $\bar{B}$
- $H(\bar{B}) = H(0) - \sum_i \mu_i \bar{B}_i$
- $H(\bar{B})^\star = H(-\bar{B})$ for the Hamiltonian
- $G(\bar{B})^\star = G(-\bar{B})$ for the Gibbs state
- $U_t(\bar{H})^\star = U_{-t}(-\bar{B})$
- therefore time reversal, however $\bar{B} \to -\bar{B}$
- Maxwell's equations
- $\epsilon_0 \boldsymbol{\nabla} \cdot \mathbf{E} = \rho$ and $(1/\mu_0)\boldsymbol{\nabla} \times \mathbf{B} - \epsilon_0 \dot{\mathbf{E}} = \mathbf{j}$
- $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ and $\boldsymbol{\nabla} \times \mathbf{E} + \dot{\mathbf{B}} = 0$

Time reversal (ctd.)

- Static magnetic induction $\bar{B}$
- $H(\bar{B}) = H(0) - \sum_i \mu_i \bar{B}_i$
- $H(\bar{B})^\star = H(-\bar{B})$ for the Hamiltonian
- $G(\bar{B})^\star = G(-\bar{B})$ for the Gibbs state
- $U_t(\bar{H})^\star = U_{-t}(-\bar{B})$
- therefore time reversal, however $\bar{B} \to -\bar{B}$
- Maxwell's equations
- $\epsilon_0 \boldsymbol{\nabla} \cdot \mathbf{E} = \rho$ and $(1/\mu_0)\boldsymbol{\nabla} \times \mathbf{B} - \epsilon_0 \dot{\mathbf{E}} = \mathbf{j}$
- $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ and $\boldsymbol{\nabla} \times \mathbf{E} + \dot{\mathbf{B}} = 0$
- $t, \mathbf{x}, \rho, \mathbf{j}, \mathbf{E}, \mathbf{B}$ go to $-t, \mathbf{x}, \rho, -\mathbf{j}, \mathbf{E}, -\mathbf{B}$

Time reversal (ctd.)

- Static magnetic induction $\bar{B}$
- $H(\bar{B}) = H(0) - \sum_i \mu_i \bar{B}_i$
- $H(\bar{B})^\star = H(-\bar{B})$ for the Hamiltonian
- $G(\bar{B})^\star = G(-\bar{B})$ for the Gibbs state
- $U_t(\bar{H})^\star = U_{-t}(-\bar{B})$
- therefore time reversal, however $\bar{B} \to -\bar{B}$
- Maxwell's equations
- $\epsilon_0 \boldsymbol{\nabla} \cdot \mathbf{E} = \rho$ and $(1/\mu_0)\boldsymbol{\nabla} \times \mathbf{B} - \epsilon_0 \dot{\mathbf{E}} = \mathbf{j}$
- $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ and $\boldsymbol{\nabla} \times \mathbf{E} + \dot{\mathbf{B}} = 0$
- $t, \mathbf{x}, \rho, \mathbf{j}, \mathbf{E}, \mathbf{B}$ go to $-t, \mathbf{x}, \rho, -\mathbf{j}, \mathbf{E}, -\mathbf{B}$
- time reversal is a symmetry

Time reversal (ctd.)

- Static magnetic induction $\bar{B}$
- $H(\bar{B}) = H(0) - \sum_i \mu_i \bar{B}_i$
- $H(\bar{B})^\star = H(-\bar{B})$ for the Hamiltonian
- $G(\bar{B})^\star = G(-\bar{B})$ for the Gibbs state
- $U_t(\bar{H})^\star = U_{-t}(-\bar{B})$
- therefore time reversal, however $\bar{B} \to -\bar{B}$
- Maxwell's equations
- $\epsilon_0 \boldsymbol{\nabla} \cdot \mathbf{E} = \rho$ and $(1/\mu_0)\boldsymbol{\nabla} \times \mathbf{B} - \epsilon_0 \dot{\mathbf{E}} = \mathbf{j}$
- $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$ and $\boldsymbol{\nabla} \times \mathbf{E} + \dot{\mathbf{B}} = 0$
- $t, \mathbf{x}, \rho, \mathbf{j}, \mathbf{E}, \mathbf{B}$ go to $-t, \mathbf{x}, \rho, -\mathbf{j}, \mathbf{E}, -\mathbf{B}$
- time reversal is a symmetry
- $\dot{\mathbf{p}} = q\{\mathbf{E} + \mathbf{v} \times \mathbf{B}\}$

Time reversal (ctd.)

- Static magnetic induction $\bar{B}$
- $H(\bar{B}) = H(0) - \sum_i \mu_i \bar{B}_i$
- $H(\bar{B})^\star = H(-\bar{B})$ for the Hamiltonian
- $G(\bar{B})^\star = G(-\bar{B})$ for the Gibbs state
- $U_t(\bar{H})^\star = U_{-t}(-\bar{B})$
- therefore time reversal, however $\bar{B} \to -\bar{B}$
- Maxwell's equations
- $\epsilon_0 \mathbf{\nabla} \cdot \mathbf{E} = \rho$ and $(1/\mu_0)\mathbf{\nabla} \times \mathbf{B} - \epsilon_0 \dot{\mathbf{E}} = \mathbf{j}$
- $\mathbf{\nabla} \cdot \mathbf{B} = 0$ and $\mathbf{\nabla} \times \mathbf{E} + \dot{\mathbf{B}} = 0$
- $t, \mathbf{x}, \rho, \mathbf{j}, \mathbf{E}, \mathbf{B}$ go to $-t, \mathbf{x}, \rho, -\mathbf{j}, \mathbf{E}, -\mathbf{B}$
- time reversal is a symmetry
- $\dot{\mathbf{p}} = q\{\mathbf{E} + \mathbf{v} \times \mathbf{B}\}$
- interaction with charged particles as well

Onsager Theorem

- $H_t = H(\bar{B}) - \sum_s \lambda_s(t) V_s$

Onsager Theorem

- $H_t = H(\bar{B}) - \sum_s \lambda_s(t) V_s$
- $\operatorname{tr} W_t V_r = \operatorname{tr} G(\bar{B}) V_r + \int_0^\infty d\tau \, \Gamma_{rs}(\bar{B};\tau) \lambda_s(t-\tau)$

Onsager Theorem

- $H_t = H(\bar{B}) - \sum_s \lambda_s(t) V_s$
- $\operatorname{tr} W_t V_r = \operatorname{tr} G(\bar{B}) V_r + \int_0^\infty d\tau\, \Gamma_{rs}(\bar{B};\tau)\lambda_s(t-\tau)$
- $\Gamma_{rs}(\bar{B};\tau) = \operatorname{tr} G(\bar{B})\, \frac{i}{\hbar}\, [U_{-\tau}(\bar{B}) V_r U_\tau(\bar{B}), V_s]$

Onsager Theorem

- $H_t = H(\bar{B}) - \sum_s \lambda_s(t) V_s$
- $\operatorname{tr} W_t V_r = \operatorname{tr} G(\bar{B}) V_r + \int_0^\infty d\tau \, \Gamma_{rs}(\bar{B};\tau) \lambda_s(t-\tau)$
- $\Gamma_{rs}(\bar{B};\tau) = \operatorname{tr} G(\bar{B}) \frac{i}{\hbar} [U_{-\tau}(\bar{B}) V_r U_\tau(\bar{B}), V_s]$
- can be rewritten as $\operatorname{tr} G(\bar{B})^\star \{ \frac{i}{\hbar} [U_{-\tau}(\bar{B}) V_r U_\tau(\bar{B}), V_s] \}^\star$

Onsager Theorem

- $H_t = H(\bar{B}) - \sum_s \lambda_s(t) V_s$
- $\operatorname{tr} W_t V_r = \operatorname{tr} G(\bar{B}) V_r + \int_0^\infty d\tau \, \Gamma_{rs}(\bar{B};\tau)\lambda_s(t-\tau)$
- $\Gamma_{rs}(\bar{B};\tau) = \operatorname{tr} G(\bar{B}) \, \frac{i}{\hbar} \, [U_{-\tau}(\bar{B}) V_r U_\tau(\bar{B}), V_s]$
- can be rewritten as $\operatorname{tr} G(\bar{B})^\star \{\frac{i}{\hbar}[U_{-\tau}(\bar{B}) V_r U_\tau(\bar{B}), V_s]\}^\star$
- $= -\operatorname{tr} G(-\bar{B}) \frac{i}{\hbar}[U_\tau(-\bar{B}) V_r^\star U_{-\tau}(-\bar{B}), V_s^\star]$

Onsager Theorem

- $H_t = H(\bar{B}) - \sum_s \lambda_s(t) V_s$
- $\text{tr}\, W_t V_r = \text{tr}\, G(\bar{B}) V_r + \int_0^\infty d\tau\, \Gamma_{rs}(\bar{B};\tau)\lambda_s(t-\tau)$
- $\Gamma_{rs}(\bar{B};\tau) = \text{tr}\, G(\bar{B})\, \frac{i}{\hbar}\, [U_{-\tau}(\bar{B}) V_r U_\tau(\bar{B}), V_s]$
- can be rewritten as $\text{tr}\, G(\bar{B})^\star \{\frac{i}{\hbar}[U_{-\tau}(\bar{B}) V_r U_\tau(\bar{B}), V_s]\}^\star$
- $= -\,\text{tr}\, G(-\bar{B}) \frac{i}{\hbar}[U_\tau(-\bar{B}) V_r^\star U_{-\tau}(-\bar{B}), V_s^\star]$
- $= -\eta_r \eta_s\, \text{tr}\, G(-\bar{B}) \frac{i}{\hbar}[U_\tau(-\bar{B}) V_r U_{-\tau}(-\bar{B}), V_s]$

Onsager Theorem

- $H_t = H(\bar{B}) - \sum_s \lambda_s(t) V_s$
- $\operatorname{tr} W_t V_r = \operatorname{tr} G(\bar{B}) V_r + \int_0^\infty d\tau\, \Gamma_{rs}(\bar{B};\tau)\lambda_s(t-\tau)$
- $\Gamma_{rs}(\bar{B};\tau) = \operatorname{tr} G(\bar{B}) \frac{i}{\hbar} [U_{-\tau}(\bar{B}) V_r U_\tau(\bar{B}), V_s]$
- can be rewritten as $\operatorname{tr} G(\bar{B})^\star \{\frac{i}{\hbar}[U_{-\tau}(\bar{B}) V_r U_\tau(\bar{B}), V_s]\}^\star$
- $= -\operatorname{tr} G(-\bar{B}) \frac{i}{\hbar}[U_\tau(-\bar{B}) V_r^\star U_{-\tau}(-\bar{B}), V_s^\star]$
- $= -\eta_r \eta_s \operatorname{tr} G(-\bar{B}) \frac{i}{\hbar}[U_\tau(-\bar{B}) V_r U_{-\tau}(-\bar{B}), V_s]$
- $= -\eta_r \eta_s \operatorname{tr} G(-\bar{B}) \frac{i}{\hbar}[V_r, U_{-\tau}(-\bar{B}) V_s] U_\tau(-\bar{B})]$

Onsager Theorem

- $H_t = H(\bar{B}) - \sum_s \lambda_s(t) V_s$
- $\text{tr}\, W_t V_r = \text{tr}\, G(\bar{B}) V_r + \int_0^\infty d\tau\, \Gamma_{rs}(\bar{B};\tau)\lambda_s(t-\tau)$
- $\Gamma_{rs}(\bar{B};\tau) = \text{tr}\, G(\bar{B})\, \frac{i}{\hbar}\, [U_{-\tau}(\bar{B}) V_r U_\tau(\bar{B}), V_s]$
- can be rewritten as $\text{tr}\, G(\bar{B})^\star \{ \frac{i}{\hbar}[U_{-\tau}(\bar{B}) V_r U_\tau(\bar{B}), V_s]\}^\star$
- $= -\,\text{tr}\, G(-\bar{B}) \frac{i}{\hbar}[U_\tau(-\bar{B}) V_r^\star U_{-\tau}(-\bar{B}), V_s^\star]$
- $= -\eta_r \eta_s\, \text{tr}\, G(-\bar{B}) \frac{i}{\hbar}[U_\tau(-\bar{B}) V_r U_{-\tau}(-\bar{B}), V_s]$
- $= -\eta_r \eta_s\, \text{tr}\, G(-\bar{B}) \frac{i}{\hbar}[V_r, U_{-\tau}(-\bar{B}) V_s] U_\tau(-\bar{B})]$
- $\Gamma_{rs}(\bar{B};\tau) = \eta_r \eta_s \Gamma_{sr}(-\bar{B};\tau)$

Onsager Theorem

- $H_t = H(\bar{B}) - \sum_s \lambda_s(t) V_s$
- $\operatorname{tr} W_t V_r = \operatorname{tr} G(\bar{B}) V_r + \int_0^\infty d\tau\, \Gamma_{rs}(\bar{B};\tau)\lambda_s(t-\tau)$
- $\Gamma_{rs}(\bar{B};\tau) = \operatorname{tr} G(\bar{B})\, \frac{i}{\hbar}\, [U_{-\tau}(\bar{B}) V_r U_\tau(\bar{B}), V_s]$
- can be rewritten as $\operatorname{tr} G(\bar{B})^\star \{\frac{i}{\hbar}[U_{-\tau}(\bar{B}) V_r U_\tau(\bar{B}), V_s]\}^\star$
- $= -\operatorname{tr} G(-\bar{B}) \frac{i}{\hbar}[U_\tau(-\bar{B}) V_r^\star U_{-\tau}(-\bar{B}), V_s^\star]$
- $= -\eta_r \eta_s \operatorname{tr} G(-\bar{B}) \frac{i}{\hbar}[U_\tau(-\bar{B}) V_r U_{-\tau}(-\bar{B}), V_s]$
- $= -\eta_r \eta_s \operatorname{tr} G(-\bar{B}) \frac{i}{\hbar}[V_r, U_{-\tau}(-\bar{B}) V_s] U_\tau(-\bar{B})]$
- $\Gamma_{rs}(\bar{B};\tau) = \eta_r \eta_s \Gamma_{sr}(-\bar{B};\tau)$
- Fourier transformation results in Onsager relations:

$$\chi_{rs}(\bar{B};\omega) = \eta_r \eta_s \chi_{sr}(-\bar{B};\omega)$$

Lars Onsager, Nobel prize 1968

Onsager relations for kinetic coefficients

- Flux of V is $J = \dot{V} = \frac{i}{\hbar}[H, V]$

Onsager relations for kinetic coefficients

- Flux of V is $J = \dot{V} = \frac{i}{\hbar}[H, V]$
- For electric polarization $\mathbf{J}(\mathbf{x}) = \dot{\mathbf{P}}(\mathbf{x}) = \sum_a q_a \dot{\mathbf{x}}_a \delta^3(\mathbf{x}_a - \mathbf{x})$

Onsager relations for kinetic coefficients

- Flux of V is $J = \dot{V} = \frac{i}{\hbar}[H, V]$
- For electric polarization $\mathbf{J}(\mathbf{x}) = \dot{\mathbf{P}}(\mathbf{x}) = \sum_a q_a \dot{\mathbf{x}}_a \delta^3(\mathbf{x}_a - \mathbf{x})$
- ... this is just the electric current density.

Onsager relations for kinetic coefficients

- Flux of V is $J = \dot{V} = \frac{i}{\hbar}[H, V]$
- For electric polarization $\mathbf{J}(\mathbf{x}) = \dot{\mathbf{P}}(\mathbf{x}) = \sum_a q_a \dot{\mathbf{x}}_a \delta^3(\mathbf{x}_a - \mathbf{x})$
- ...this is just the electric current density.
- $\dot{V}_r(\tau) = dV_r(\tau)/d\tau$

Onsager relations for kinetic coefficients

- Flux of V is $J = \dot{V} = \frac{i}{\hbar}[H, V]$
- For electric polarization $\mathbf{J}(\mathbf{x}) = \dot{\mathbf{P}}(\mathbf{x}) = \sum_a q_a \dot{\mathbf{x}}_a \delta^3(\mathbf{x}_a - \mathbf{x})$
- ...this is just the electric current density.
- $\dot{V}_r(\tau) = dV_r(\tau)/d\tau$
- We conclude

$$J_r(t) = < J_r >_t = \sum_s \int_0^\infty d\tau \, \dot{\Gamma}_{rs}(\tau) \lambda_s(t - \tau)$$

Onsager relations for kinetic coefficients

- Flux of V is $J = \dot{V} = \frac{i}{\hbar}[H, V]$
- For electric polarization $\mathbf{J}(\mathbf{x}) = \dot{\mathbf{P}}(\mathbf{x}) = \sum_a q_a \dot{\mathbf{x}}_a \delta^3(\mathbf{x}_a - \mathbf{x})$
- ...this is just the electric current density.
- $\dot{V}_r(\tau) = dV_r(\tau)/d\tau$
- We conclude

$$J_r(t) = < J_r >_t = \sum_s \int_0^\infty d\tau\, \dot{\Gamma}_{rs}(\tau)\lambda_s(t-\tau)$$

- Fourier tansform $\hat{J}_r(\omega) = \sum_s \sigma_{rs}(\omega)\hat{\lambda}_s(\omega)$

Onsager relations for kinetic coefficients

- Flux of V is $J = \dot{V} = \frac{i}{\hbar}[H, V]$
- For electric polarization $\mathbf{J}(\mathbf{x}) = \dot{\mathbf{P}}(\mathbf{x}) = \sum_a q_a \dot{\mathbf{x}}_a \delta^3(\mathbf{x}_a - \mathbf{x})$
- ...this is just the electric current density.
- $\dot{V}_r(\tau) = dV_r(\tau)/d\tau$
- We conclude

$$J_r(t) = < J_r >_t = \sum_s \int_0^\infty d\tau\, \dot{\Gamma}_{rs}(\tau) \lambda_s(t - \tau)$$

- Fourier tansform $\hat{J}_r(\omega) = \sum_s \sigma_{rs}(\omega) \hat{\lambda}_s(\omega)$
- ...where σ_{rs} is the Fourier transform of $\theta(\tau)\, \dot{\Gamma}_{rs}(\tau)$

Onsager relations for kinetic coefficients

- Flux of V is $J = \dot{V} = \frac{i}{\hbar}[H, V]$
- For electric polarization $\mathbf{J}(\mathbf{x}) = \dot{\mathbf{P}}(\mathbf{x}) = \sum_a q_a \dot{\mathbf{x}}_a \delta^3(\mathbf{x}_a - \mathbf{x})$
- ...this is just the electric current density.
- $\dot{V}_r(\tau) = dV_r(\tau)/d\tau$
- We conclude

$$J_r(t) =< J_r >_t = \sum_s \int_0^\infty d\tau\, \dot{\Gamma}_{rs}(\tau)\lambda_s(t-\tau)$$

- Fourier tansform $\hat{J}_r(\omega) = \sum_s \sigma_{rs}(\omega)\hat{\lambda}_s(\omega)$
- ...where σ_{rs} is the Fourier transform of $\theta(\tau)\,\dot{\Gamma}_{rs}(\tau)$
- Onsger relations for kinetic coefficients

$$\sigma_{rs}(\bar{B};\omega) = \sigma_{sr}(-\bar{B};\omega)$$

Ohm's law and Hall effect

- If the equilibrium state of matter is disturbed by an oscillating electric field, there will be an electric current

Ohm's law and Hall effect

- If the equilibrium state of matter is disturbed by an oscillating electric field, there will be an electric current
- $J_i(\omega) = \sum_j \sigma_{ij}(\bar{B};\omega)\, E_j(\omega)$

Ohm's law and Hall effect

- If the equilibrium state of matter is disturbed by an oscillating electric field, there will be an electric current
- $J_i(\omega) = \sum_j \sigma_{ij}(\bar{B};\omega)\, E_j(\omega)$
- This is Ohm's law

Ohm's law and Hall effect

- If the equilibrium state of matter is disturbed by an oscillating electric field, there will be an electric current
- $J_i(\omega) = \sum_j \sigma_{ij}(\bar{B};\omega)\, E_j(\omega)$
- This is Ohm's law
- $\sigma_{ij}(\bar{B};\omega) = \sigma_{ij}(\omega) + \sum_k H_{ijk}(\omega) B_k + \ldots$

Ohm's law and Hall effect

- If the equilibrium state of matter is disturbed by an oscillating electric field, there will be an electric current
- $J_i(\omega) = \sum_j \sigma_{ij}(\bar{B};\omega)\, E_j(\omega)$
- This is Ohm's law
- $\sigma_{ij}(\bar{B};\omega) = \sigma_{ij}(\omega) + \sum_k H_{ijk}(\omega) B_k + \dots$
- σ_{ij} is symmetric

Ohm's law and Hall effect

- If the equilibrium state of matter is disturbed by an oscillating electric field, there will be an electric current
- $J_i(\omega) = \sum_j \sigma_{ij}(\bar{B};\omega)\, E_j(\omega)$
- This is Ohm's law
- $\sigma_{ij}(\bar{B};\omega) = \sigma_{ij}(\omega) + \sum_k H_{ijk}(\omega) B_k + \dots$
- σ_{ij} is symmetric
- H_{ijk} is <u>antisymmetric</u> for $i \leftrightarrow j$

Ohm's law and Hall effect

- If the equilibrium state of matter is disturbed by an oscillating electric field, there will be an electric current
- $J_i(\omega) = \sum_j \sigma_{ij}(\bar{B};\omega)\, E_j(\omega)$
- This is Ohm's law
- $\sigma_{ij}(\bar{B};\omega) = \sigma_{ij}(\omega) + \sum_k H_{ijk}(\omega) B_k + \dots$
- σ_{ij} is symmetric
- H_{ijk} is antisymmetric for $i \leftrightarrow j$
- $H_{ijk}(\omega) = H(\omega)\epsilon_{ijk}$

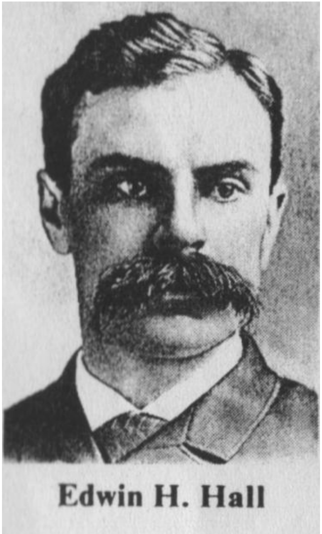
Ohm's law and Hall effect

- If the equilibrium state of matter is disturbed by an oscillating electric field, there will be an electric current
- $J_i(\omega) = \sum_j \sigma_{ij}(\bar{B};\omega)\, E_j(\omega)$
- This is Ohm's law
- $\sigma_{ij}(\bar{B};\omega) = \sigma_{ij}(\omega) + \sum_k H_{ijk}(\omega) B_k + \dots$
- σ_{ij} is symmetric
- H_{ijk} is antisymmetric for $i \leftrightarrow j$
- $H_{ijk}(\omega) = H(\omega)\epsilon_{ijk}$
- Additional term describes the Hall effect:

Ohm's law and Hall effect

- If the equilibrium state of matter is disturbed by an oscillating electric field, there will be an electric current
- $J_i(\omega) = \sum_j \sigma_{ij}(\bar{B};\omega)\, E_j(\omega)$
- This is Ohm's law
- $\sigma_{ij}(\bar{B};\omega) = \sigma_{ij}(\omega) + \sum_k H_{ijk}(\omega) B_k + \dots$
- σ_{ij} is symmetric
- H_{ijk} is <u>antisymmetric</u> for $i \leftrightarrow j$
- $H_{ijk}(\omega) = H(\omega)\epsilon_{ijk}$
- Additional term describes the Hall effect:
- $\mathbf{J}(\omega) = H(\omega)\,\mathbf{E}(\omega) \times \bar{\mathbf{B}}$

Georg Simon Ohm



Edwin Herbert Hall