Optical Activity

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Roadmap

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- Permittivity

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- No external fields

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- note that $\tilde{\boldsymbol{E}}$, $\tilde{\boldsymbol{H}}$ and permittivity ϵ are Fourier transforms and depend on $(\omega, \boldsymbol{q})$.

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- in general, permittivity ϵ_{ij} depends on angular frequency ω and wave vector $\boldsymbol{q}$

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- $G_{ij}(\tau, \boldsymbol{\xi}) \approx G_{ij}(\tau)\, \delta^3(\boldsymbol{\xi})$

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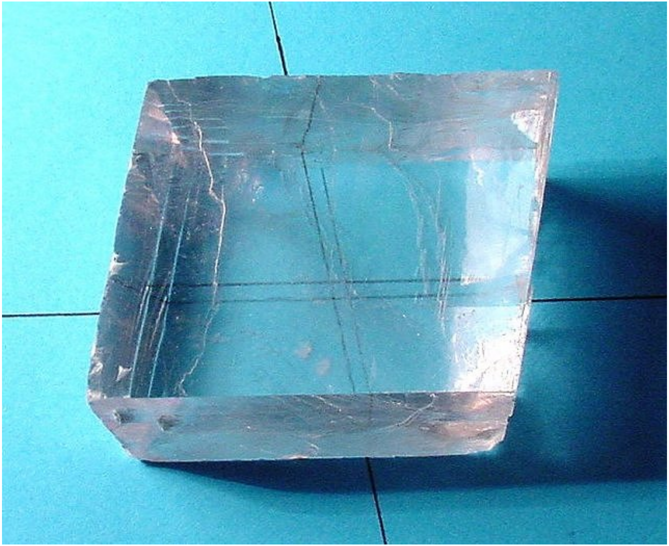

Double refraction (birefringence) by calcite

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- effect is fast (GHz), but requires large field strength
- therefore µm-optics (Integrated Optics)

A commercial Pockels cell for modulating light

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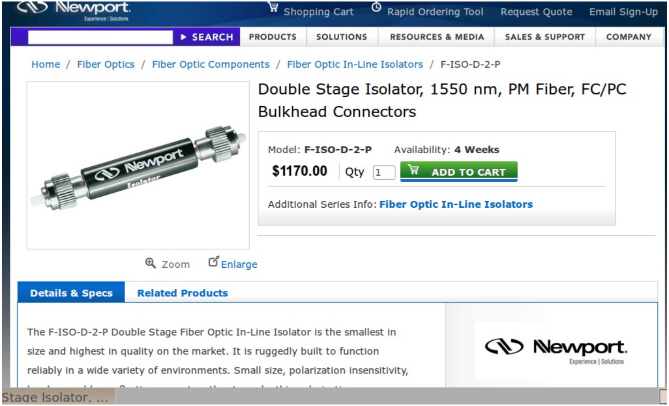
- goal: realize the optical isolator in µm-optics



A commercial optical isolator

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- G_{kl} is a rank 2 pseudo-tensor

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- $\det R = +1$: proper rotation

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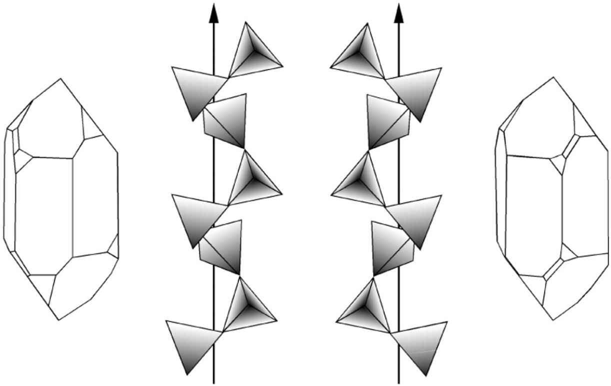

Right vs. left handed quartz crystals.

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- Question: Are all sugar producing plants copies of the first plant, which randomly decided between left and right?

Thomas Seebeck, German physicist, 1770-1831

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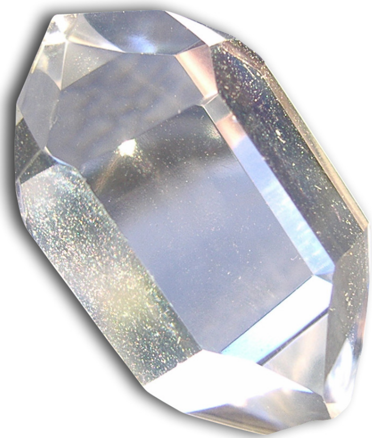

Natural quartz