

Optical Activity

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Lecture presented at APS, Nankai University, China

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October/November 2011

Roadmap

Permittivity

No external
fields

External
electric field

External
magnetic field

Optical
activity

Roadmap

Roadmap

- Permittivity

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- No external fields

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- External electric field

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- External magnetic field
- Optical activity

The electromagnetic field

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- action on charged particles

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- Maxwell's equations with $\varrho = 0$, $\mathbf{j} = 0$, $\mu = 1$

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- note that $\tilde{\mathbf{E}}$, $\tilde{\mathbf{H}}$ and permittivity ϵ are Fourier transforms and depend on (ω, \mathbf{q}) .

Permittivity

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- most general causal linear relationship between electrical field and polarization field

$$P_i(t, \mathbf{x}) = \epsilon_0 \int_0^\infty d\tau \int d^3\xi G_{ij}(\tau, \boldsymbol{\xi}) E_j(t - \tau, \mathbf{x} - \boldsymbol{\xi})$$

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- in general, permittivity ϵ_{ij} depends on angular frequency ω and wave vector \mathbf{q}

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- $G_{ij}(\tau, \boldsymbol{\xi}) \approx G_{ij}(\tau) \delta^3(\boldsymbol{\xi})$

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- at least difficult

Isotropic and birefringent media

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- no external electric or magnetic field

Isotropic and birefringent media

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Roadmap

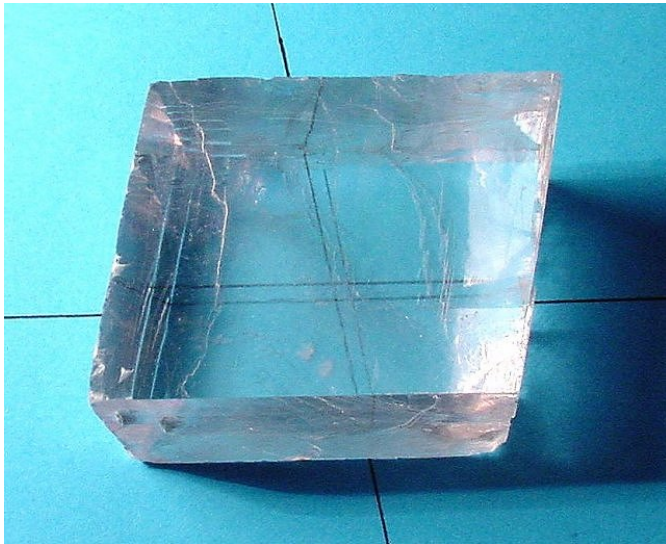
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Double refraction (birefringence) by calcite

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- therefore μm -optics (Integrated Optics)



A commercial Pockels cell for modulating light

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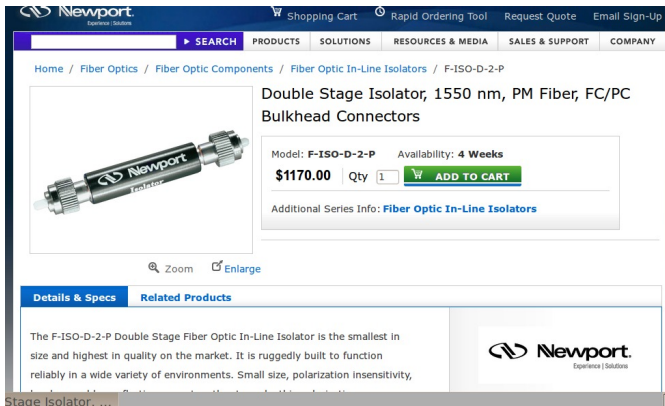
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- goal: realize the optical isolator in μm -optics



The screenshot displays the Newport website's product page for a 'Double Stage Isolator, 1550 nm, PM Fiber, FC/PC Bulkhead Connectors'. The page features a navigation bar with links to 'Shopping Cart', 'Rapid Ordering Tool', 'Request Quote', and 'Email Sign-Up'. Below the navigation bar is a search bar and a menu with categories like 'PRODUCTS', 'SOLUTIONS', 'RESOURCES & MEDIA', 'SALES & SUPPORT', and 'COMPANY'. The main content area includes a breadcrumb trail: 'Home / Fiber Optics / Fiber Optic Components / Fiber Optic In-Line Isolators / F-ISO-D-2-P'. A large image of the isolator is shown on the left, with 'Newport Isolator' printed on it. To the right of the image, the product title is 'Double Stage Isolator, 1550 nm, PM Fiber, FC/PC Bulkhead Connectors'. Below the title, the model is 'F-ISO-D-2-P' and the availability is '4 Weeks'. The price is '\$1170.00' and the quantity is '1'. There is an 'ADD TO CART' button. Below the price, it says 'Additional Series Info: Fiber Optic In-Line Isolators'. At the bottom of the page, there is a 'Details & Specs' section and a 'Related Products' section. The 'Details & Specs' section contains text about the isolator's size and quality. The 'Related Products' section is currently empty.

Newport
Experience | Solutions

Shopping Cart Rapid Ordering Tool Request Quote Email Sign-Up

SEARCH PRODUCTS SOLUTIONS RESOURCES & MEDIA SALES & SUPPORT COMPANY

Home / Fiber Optics / Fiber Optic Components / Fiber Optic In-Line Isolators / F-ISO-D-2-P

Double Stage Isolator, 1550 nm, PM Fiber, FC/PC Bulkhead Connectors

Model: **F-ISO-D-2-P** Availability: **4 Weeks**

\$1170.00 Qty [ADD TO CART](#)

Additional Series Info: [Fiber Optic In-Line Isolators](#)

Zoom Enlarge

Details & Specs **Related Products**

The F-ISO-D-2-P Double Stage Fiber Optic In-Line Isolator is the smallest in size and highest in quality on the market. It is ruggedly built to function reliably in a wide variety of environments. Small size, polarization insensitivity,

Newport
Experience | Solutions

Stage Isolator, ...

A commercial optical isolator

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I found this in the internet when looking for *optical activity*.

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- $\Delta\epsilon_{ij}^{\text{oa}} = i\epsilon_{ijk} g_k$ with $g_k = G_{kl} q_l$

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Optical activity

Roadmap

Permittivity

No external
fieldsExternal
electric fieldExternal
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Genuine and pseudo tensors

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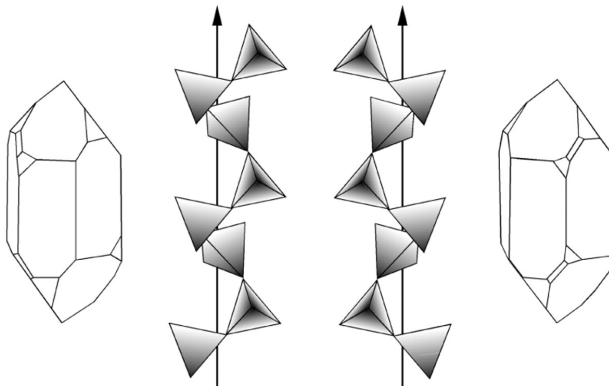
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Right vs. left handed quartz crystals.

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- Question: Are all sugar producing plants copies of the first plant, which randomly decided between left and right?



Thomas Seebeck, German physicist, 1770-1831

Quartz

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Quartz

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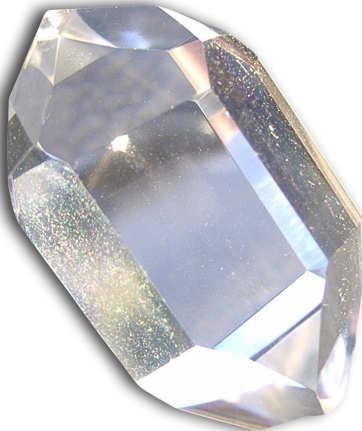
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Natural quartz