Peter Hertel Overview Terminology Simple model Solution Discussion

Photonic Crystals

Photonic Crystals

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Overview

- Terminology
- Simple mod
- Solution
- Discussion
- Applications

- Photonic Crystals are metamaterials
- with strange optical properties
- structures with a periodical permittivity
- there is an optical band gap
- we study a very simple model
- the transmission coefficient vanishes in an entire frequency range !
- remarks on more complex structures
- remarks on applications

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Overview

Terminology

Simple mode

Solution

Discussion

Applications

Graduate Texts in Physic

Continuum Physic

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The out of balance equations for context, flow and production of particles, mans, charge, measureman, energy of attrapped in againment of materials, or constitutive equations. They describe order classes of materials, each no struct Balad and gause, detects findeenties or electrical conductorus. We discuss the response of matter to arapidity materials granular parameters, in particular the descript field structure participation transmoved of attaintic if thermodynamics. An appendix on fields and a glossary record of this hadd-of open one occentisment physics.

Students of physics, explorering and ruleud fields will boundit from the clear presentation of worked examples and the variety of solution methods, including transmittal bechiliques. Lectures or advanced tradents may predic from the unified view on a substantial part of physics. It may help them to embed their research field conceptually while a wider contents. **Graduate Texts in Physics**

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Continuum Physics

 $\langle 2 \rangle$

Continuum Physics

Will appear soon. Contains a subsection within a section on metamaterials



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- photonic crystals are crystals because they are periodic
- they are called photonic because the periodicity constant is comparable with the wavelength of photons (in the visible or near infrared)
- ordinary crystals should be called X ray crystals
- although generally classified as metamaterials (or man-made), this is not true
- photonic crystals appear in nature
- the permittivity is that of ordinary matter
- only the periodic structure is man-made

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The iridescence of the 'eyes' comes from a regular array of layers with different refractive indexes. To be distinguished from color effects by pigments.

Simple model

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- one-dimensional, two kinds of slabs
- unit cell is region -a/2 < x < a/2
- refractive index within -d/2 < x < d/2 is n_2
- outside it is n_1
- this unit cell is ${\cal N}$ times translated by the lattice constant a
- electric field

$$\boldsymbol{E}(t,x) = \begin{pmatrix} 0\\ E(x)\\ 0 \end{pmatrix} e^{-\mathrm{i}\omega t}$$

• obeys mode wave equation $E''(x) + k_0^2 \epsilon(x) E(x) = 0 \text{ with } k_0 = \frac{\omega}{-1}$

Propagation transfer matrix

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- section of constant permittivity $\epsilon = n^2$ $E(x) = a_{+} e^{+ink_{0}x} + a_{-} e^{-ink_{0}x}$
- propagation by distance y is described by $E(x+y) = \bar{a}_{+} e^{+ink_{0}x} + \bar{a}_{-} e^{-ink_{0}x}$
- where

$$\begin{pmatrix} \bar{a}_+\\ \bar{a}_- \end{pmatrix} = P(n,y) \begin{pmatrix} a_+\\ a_- \end{pmatrix}$$

with

$$P(n,y) = \left(\begin{array}{cc} e^{+\mathrm{i}nk_0y} & 0\\ 0 & e^{-\mathrm{i}nk_0y} \end{array}\right)$$

• propagation transfer matrix P is diagonal

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Discontinuity transfer matrix

- transition through a discontinuity from $n^{\,\prime}$ to $n^{\prime\prime}$
- transversal components $E_y=E$ and $H_z\propto E\,'$ must be continuous
- again

$$\left(\begin{array}{c} \bar{a}_+\\ \bar{a}_- \end{array}\right) = D(n',n'') \left(\begin{array}{c} a_+\\ a_- \end{array}\right)$$

• where

$$D(n',n'') = \frac{1}{2n''} \left(\begin{array}{cc} n''+n' & n''-n' \\ n''-n' & n''+n' \end{array} \right)$$

- note D(n,n) = I
- also note that D is symmetric
- work it out: $D(n^{\,\prime},n^{\,\prime\prime})\,D(n^{\,\prime\prime},n^{\,\prime})=I$

Transfer matrix

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- following the propagation, transfer matrices multiply
- propagation through the unit cell
- first margin m = (d-a)/2 with n_1 , then discontinuity $n1 \to n_2$, then slab d with n_2 , then discontinuity $n_2 \to n_1$, then margin
- i.e.

 $T = P(m, n_1) D(n_1, n_2) P(d, n_2) D(n_2, n_1) P(m, n_1)$

- transfer through crystal of N unit cells is described by $T_N = T^N \label{eq:relation}$

Transmission coefficient

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• Let us solve

$$T_N\left(\begin{array}{c}0\\a_{\rm t}\end{array}\right) = \left(\begin{array}{c}a_{\rm r}\\a_{\rm i}\end{array}\right)$$

- an incident wave is impinging on the right, producing a reflected wave and a transmitted wave leaving the crystal at the left.
- transmission coefficient is

$$t = \frac{|a_{\rm t}|^2}{|a_{\rm i}|^2}$$

- reflection coefficient r defined likewise
- since T_N is unitary,

$$r+t=1$$

```
function exmmfig5
 Photonic
 Crystals
           om=linspace(0,2*pi,256);
           trans4=zeros(size(om));
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           trans16=zeros(size(om));
           for k=1:length(om)
               trans4(k)=transmission(1,1.5,0.5,4,om(k));
               trans16(k)=transmission(1,1.5,0.5,16,om(k));
Solution
           end;
           plot(om,trans4,'-r',om,trans16,'-b','linewidth',1.2);
           axis([0 2*pi 0 1]);
           print -depsc exmmfig5.eps
           end % main function photonic
           function tr=transmission(n1,n2,d,N,omega)
           P1=[exp(1i*omega*n1*(1-d)/2),0;0,exp(-1i*omega*n1*(1-d)/2)]
           P2=[exp(1i*omega*n2*d),0;0,exp(-1i*omega*n2*d)];
           D12=0.5*[1+n1/n2,1-n1/n2;1-n1/n2,1+n1/n2];
           D21=0.5*[1+n2/n1,1-n2/n1;1-n2/n1,1+n2/n1];
           T=P1*D12*P2*D21*P1;
           amp=T^N*[0;1];
           tr=1/abs(amp(2))^2;
           end % function transmission
```



Transmission coefficient t vs. $a\omega/c = k_0 a = a/2\pi\lambda$. For four unit cells (red) there are marked dips. With 16 unit cells (blue), the band gaps are clearly visible.



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- For suitable parameters, there are band gaps, that is ...
- frequency regions where light cannon propagate within a photonic crystal
- transmission or reflection coefficients vary rapidly in allowed bands
- recall iridescence
- already small crystals (N = 16) show pronounced effects
- advantage: micro-meter structures required only
- disadvantage: no magnetic properties

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Applications



Recall: the iridescence of the 'eyes' comes from a regular array of layers with different refractive indexes. To be distinguished from color effects by pigments.



Applications



Holes produces a band gap which confines light because of total reflection.





A three dimensional photonic crystal suitable for 1.5 μm infrared light.



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Screenshot of photonic crystal design and simulation software.