Photonic Crystals

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Overview

- Photonic Crystals are metamaterials
- with strange optical properties
- structures with a periodical permittivity
- there is an optical band gap
- we study a very simple model
- the transmission coefficient vanishes in an entire frequency range !
- remarks on more complex structures
- remarks on applications

Terminology

- photonic crystals are crystals because they are periodic
- they are called photonic because the periodicity constant is comparable with the wavelength of photons (in the visible or near infrared)
- ordinary crystals should be called X ray crystals
- although generally classified as metamaterials (or man-made), this is not true
- photonic crystals appear in nature
- the permittivity is that of ordinary matter
- only the periodic structure is man-made

The iridescence of the 'eyes' comes from a regular array of layers with different refractive indexes. To be distinguished from color effects by pigments.

Simple model

- one-dimensional, two kinds of slabs
- unit cell is region $-a/2 < x < a/2$
- refractive index within $-d/2 < x < d/2$ is n_2
- outside it is n_1
- this unit cell is N times translated by the lattice constant a
- electric field

$$\boldsymbol{E}(t,x) = \begin{pmatrix} 0 \\ E(x) \\ 0 \end{pmatrix} \mathrm{e}^{-\mathrm{i}\omega t}$$

- obeys mode wave equation

$$E''(x) + k_0^2\,\epsilon(x)\,E(x) = 0 \text{ with } k_0 = \frac{\omega}{c}$$

Propagation transfer matrix

- section of constant permittivity $\epsilon = n^2$

  $$E(x) = a_+ \,\mathrm{e}^{+\mathrm{i}nk_0x} + a_- \,\mathrm{e}^{-\mathrm{i}nk_0x}$$

- propagation by distance y is described by

  $$E(x+y) = \bar{a}_+ \,\mathrm{e}^{+\mathrm{i}nk_0x} + \bar{a}_- \,\mathrm{e}^{-\mathrm{i}nk_0x}$$

- where

  $$\begin{pmatrix} \bar{a}_+ \\ \bar{a}_- \end{pmatrix} = P(n,y) \begin{pmatrix} a_+ \\ a_- \end{pmatrix}$$

- with

  $$P(n,y) = \begin{pmatrix} \mathrm{e}^{+\mathrm{i}nk_0y} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}nk_0y} \end{pmatrix}$$

- propagation transfer matrix P is diagonal

Discontinuity transfer matrix

- transition through a discontinuity from n' to n''
- transversal components $E_y = E$ and $H_z \propto E'$ must be continuous
- again
$$\begin{pmatrix} \bar{a}_+ \\ \bar{a}_- \end{pmatrix} = D(n', n'') \begin{pmatrix} a_+ \\ a_- \end{pmatrix}$$
- where
$$D(n', n'') = \frac{1}{2n''} \begin{pmatrix} n'' + n' & n'' - n' \\ n'' - n' & n'' + n' \end{pmatrix}$$
- note $D(n, n) = I$
- also note that D is symmetric
- work it out: $D(n', n'')\, D(n'', n') = I$

Transfer matrix

- following the propagation, transfer matrices multiply
- propagation through the unit cell
- first margin $m = (d - a)/2$ with n_1, then discontinuity $n1 \to n_2$, then slab d witn n_2, then discontinuity $n_2 \to n_1$, then margin
- i.e.

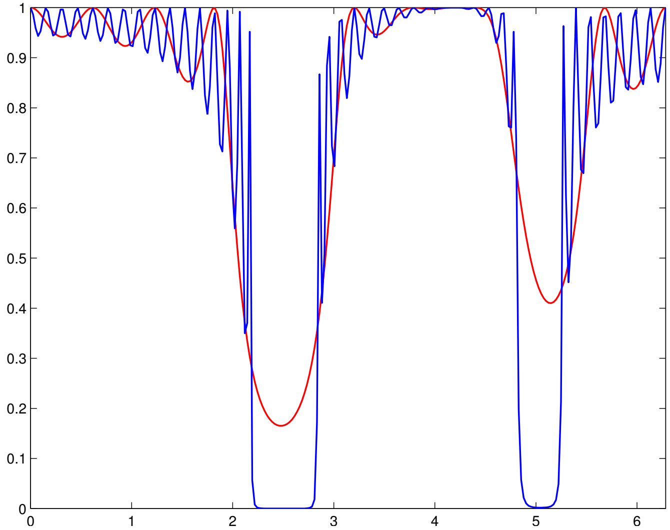
  $T = P(m, n_1)\, D(n_1, n_2)\, P(d, n_2)\, D(n_2, n_1)\, P(m, n_1)$
- transfer through crystal of N unit cells is described by $T_N = T^N$

Transmission coefficient

- Let us solve
$$T_N \begin{pmatrix} 0 \\ a_{\mathrm{t}} \end{pmatrix} = \begin{pmatrix} a_{\mathrm{r}} \\ a_{\mathrm{i}} \end{pmatrix}$$
- an incident wave is impinging on the right, producing a reflected wave and a transmitted wave leaving the crystal at the left.
- transmission coefficient is
$$t = \frac{|a_{\mathrm{t}}|^2}{|a_{\mathrm{i}}|^2}$$
- reflection coefficient r defined likewise
- since T_N is unitary,
$$r + t = 1$$

```
function exmmfig5
om=linspace(0,2*pi,256);
trans4=zeros(size(om));
trans16=zeros(size(om));
for k=1:length(om)
    trans4(k)=transmission(1,1.5,0.5,4,om(k));
    trans16(k)=transmission(1,1.5,0.5,16,om(k));
end;
plot(om,trans4,'-r',om,trans16,'-b','linewidth',1.2);
axis([0 2*pi 0 1]);
print -depsc exmmfig5.eps
end % main function photonic

function tr=transmission(n1,n2,d,N,omega)
P1=[exp(1i*omega*n1*(1-d)/2),0;0,exp(-1i*omega*n1*(1-d)/2)
P2=[exp(1i*omega*n2*d),0;0,exp(-1i*omega*n2*d)];
D12=0.5*[1+n1/n2,1-n1/n2;1-n1/n2,1+n1/n2];
D21=0.5*[1+n2/n1,1-n2/n1;1-n2/n1,1+n2/n1];
T=P1*D12*P2*D21*P1;
amp=T^N*[0;1];
tr=1/abs(amp(2))^2;
end % function transmission
```

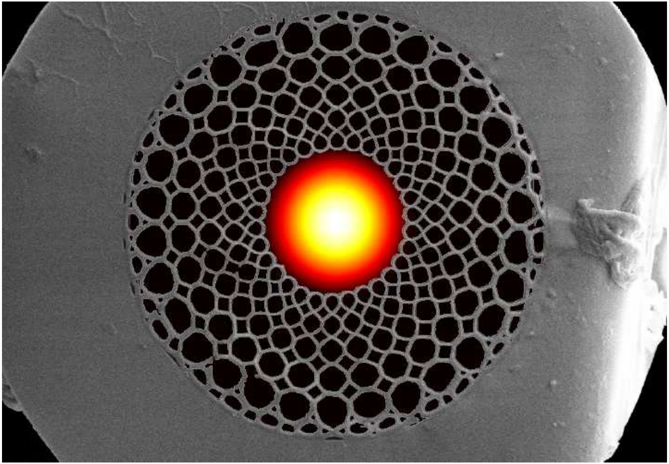


Transmission coefficient t vs. $a\omega/c = k_0 a = a/2\pi\lambda$. For four unit cells (red) there are marked dips. With 16 unit cells (blue), the band gaps are clearly visible.

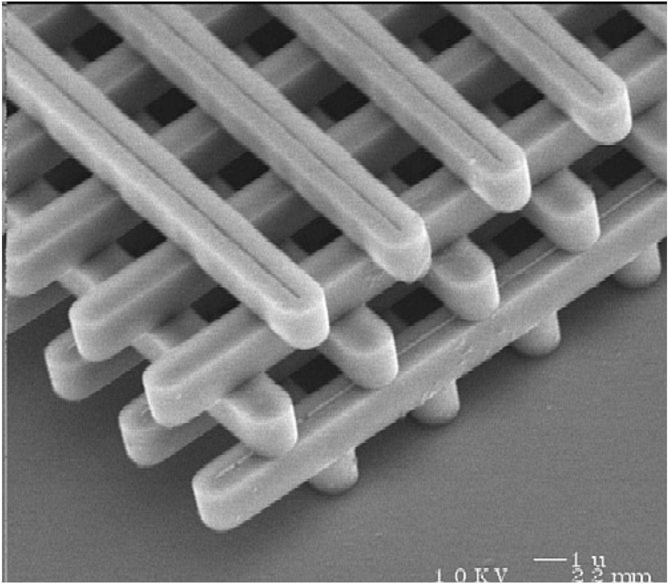
Discussion

- For suitable parameters, there are band gaps, that is ...
- frequency regions where light cannon propagate within a photonic crystal
- transmission or reflection coefficients vary rapidly in allowed bands
- recall iridescence
- already small crystals $(N = 16)$ show pronounced effects
- advantage: micro-meter structures required only
- disadvantage: no magnetic properties

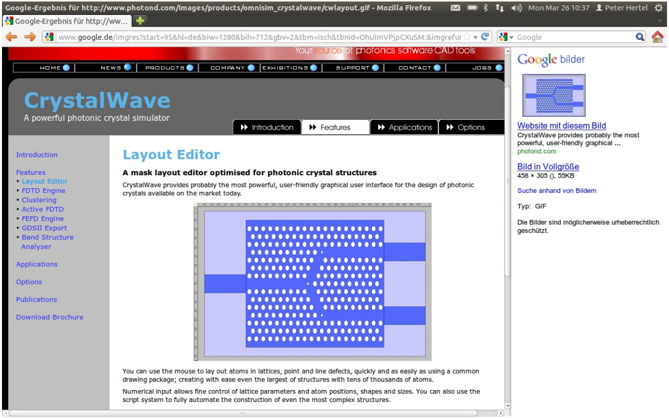
Recall: the iridescence of the 'eyes' comes from a regular array of layers with different refractive indexes. To be distinguished from color effects by pigments.



Holes produces a band gap which confines light because of total reflection.



A three dimensional photonic crystal suitable for 1.5 μm infrared light.



Screenshot of photonic crystal design and simulation software.