

Planar Waveguides

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Modes

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- all fields are of the form

$$F(t, x, y, z) = F(x) e^{i\beta z} e^{-i\omega t}$$

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TE modes

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Graded index waveguide

- Cover (air) and Substrate (glass, lithium niobate)

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- field assisted ion exchange (silver in glass)

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- permittivity profile

$$\epsilon(x) = \begin{cases} \epsilon_c & \text{for } x < 0 \\ \epsilon_s + \Delta\epsilon e^{-(x/W)^2} & \text{for } x > 0 \end{cases}$$

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- gaussian diffusion profile

```
1  LAMBDA=0.6328;
2  k0=2*pi/LAMBDA;
3  EC=1.000;
4  ES=4.800;
5  ED=0.045;
6  W=4.00;
7  xmin=-1.0;
8  xmax=4*W;
9  h=0.1;
10 x=(xmin:h:xmax)';
11 dim=size(x,1);
12 prm=(x<0).*EC+(x>=0).*(ES+ED*exp(-(x/W).^2));
13 next=ones(dim-1,1)/h^2/k0^2;
14 main=-2.0*ones(dim,1)/h^2/k0^2+prm;
15 L=diag(next,-1)+diag(main,0)+diag(next,1);
16 [evec, eval]=eig(L);
17 eff_eps=diag(eval);
18 guided=evec(:,eff_eps>ES);
19 plot(x,guided);
```

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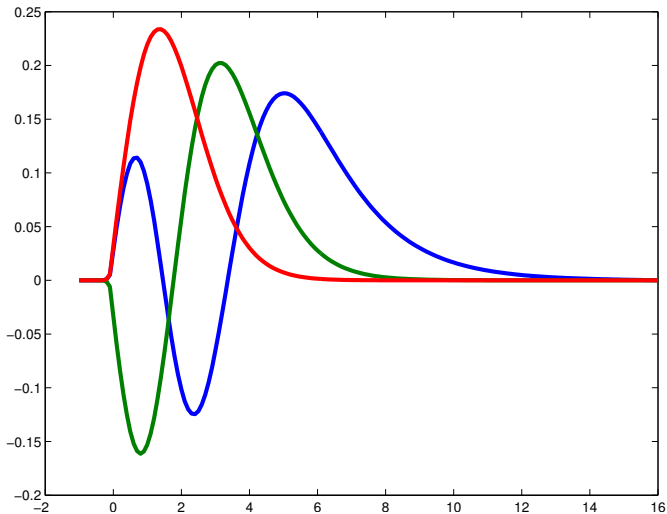
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The guided modes of a graded index waveguide.

Slab waveguides

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- one or more films of enhanced permittivities

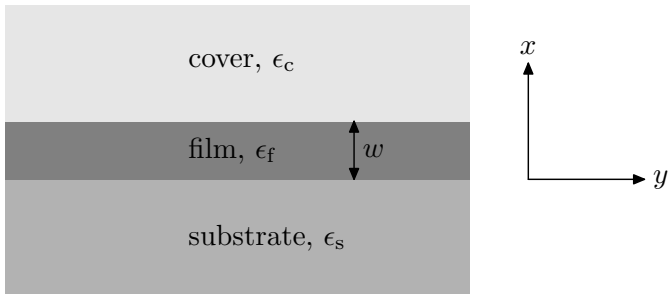
Slab waveguides

- A slabwaveguide consists of a substrate
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Slab waveguides

- A slabwaveguide consists of a substrate
- one or more films of enhanced permittivities
- and is covered by a layer of low permittivity
- the permittivity profile

$$\epsilon(x) = \begin{cases} \epsilon_s & \text{for } x < 0 \\ \epsilon_f & \text{for } 0 < x < w \\ \epsilon_c & \text{for } w < x \end{cases}$$



A slab waveguide. A film of increased permittivity on top of a substrate being covered by material of low permittivity. w is the film thickness.

Analytical solution I

- substrate:

$$E = e^{\kappa_s X} \quad \text{where } \kappa_s = k_0 \sqrt{\epsilon_{\text{eff}} - \epsilon_s}$$

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$$E = c \cos k_f x + s \sin k_f x \text{ where } k_f = k_0 \sqrt{\epsilon_f - \epsilon_{\text{eff}}}$$

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- E and E' continuous at $x = w$, **b must vanish**

Analytical solution II

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$$\cot k_f w = \frac{k_f^2 - \kappa_s \kappa_c}{k_f (\kappa_s + \kappa_c)}$$

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- $\bar{\kappa}_c = \kappa_c / \epsilon_c$, $\bar{k}_f = k_f / \epsilon_f$, $\bar{\kappa}_s = \kappa_s / \epsilon_s$

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- $\bar{\kappa}_c = \kappa_c / \epsilon_c$, $\bar{k}_f = k_f / \epsilon_f$, $\bar{\kappa}_s = \kappa_s / \epsilon_s$
- example: $n_f = 1.52$, $w_f = 1.8 \mu\text{m}$, $n_s = 1.49$, $n_c = 1.00$,
 $\lambda = 632.8 \text{ nm}$.

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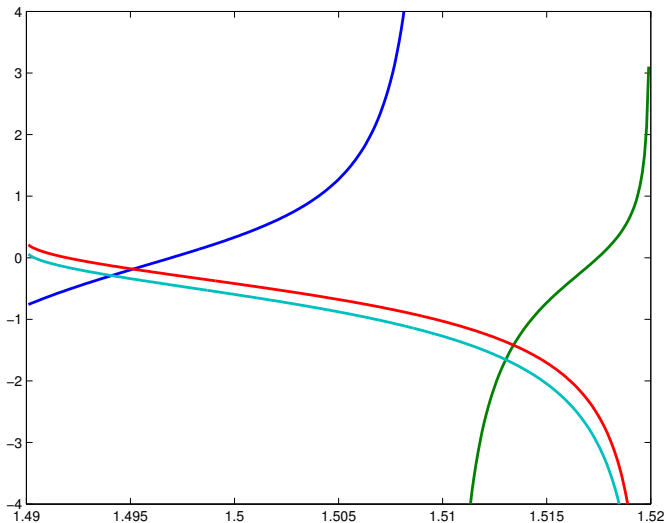
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Graphical solution for TE and TM mode conditions. Blue and green are the branches of the cotangent if plotted versus the effective index. Red and teal represent the right hand sides for TE and TM modes, respectively.

Java applet

`http://wwwhome.math.utwente.nl/~hammerm/oms.html`

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