TM modes on metal surfaces

We consider a plane interface between a dielectric cover, air say, and a good conductor, such as gold. The classical free electron gas model results in a permittivity the real part of which, well below the plasma resonance frequency, is large and negative. We show that there is at most one TM polarized surface mode, but no TE counterpart. Any medium the permittivity of which has a large negative real and a small imaginary part behaves like this, the underlying mechanism must not be surface plasmon polaritons.

The Drude model

An external electric field $\boldsymbol{E}$ will exert the following force on an electron at $\boldsymbol{x}$:

$$m\ddot{\boldsymbol{x}} = -m\gamma\dot{\boldsymbol{x}} - m\Omega^2\boldsymbol{x} - e\boldsymbol{E}(t)\,. \tag{1}$$

m and $-e$ are its mass and charge, $\boldsymbol{x}$ denotes the displacement from the equilibrium position. γ describes friction and $m\Omega^2$ is the spring constant.

We Fourier transform this equation and obtain

$$(-\omega^2 - \mathrm{i}\gamma\omega + \Omega^2)\,\hat{\boldsymbol{x}}(\omega) = -\frac{e}{m}\hat{\boldsymbol{E}}(\omega)\,. \tag{2}$$

If there are N such electrons per unit volume, the polarization is

$$\hat{\boldsymbol{P}}(\omega) = \frac{Ne^2}{m}\frac{1}{-\omega^2 - \mathrm{i}\gamma\omega + \Omega^2}\,\hat{\boldsymbol{E}}(\omega)\,. \tag{3}$$

The relative permittivity, as defined by $\hat{\boldsymbol{D}}(\omega) = \epsilon(\omega)\,\epsilon_0\,\hat{\boldsymbol{E}}(\omega)$, thus is given by

$$\epsilon(\omega) = \epsilon_\infty - \frac{\omega_\mathrm{p}^2}{\omega^2 + \mathrm{i}\gamma\omega - \Omega^2} \quad \text{where} \quad \omega_\mathrm{p}^2 = \frac{Ne^2}{m\epsilon_0}\,. \tag{4}$$

The free electron gas is characterized by $\Omega = 0$: there is no elastic binding force. ω_p then is the plasma resonance frequency. In order to take other contributions into account, the one in (4) was changed to ϵ_∞.

Maxwell equations and surface modes

We denote by x the coordinate perpendicular to the plane. $x > 0$ describes the cover (subscript c), $x < 0$ the metal (subscript m). Without loss of generality the direction of propagation can be chosen as the z axis. All components F of the electromagnetic field $\boldsymbol{E}$, $\boldsymbol{H}$ are of the form[1]

$$F(t, x, y, z) = F(x)\,\mathrm{e}^{\,\mathrm{i}\beta z}\;\mathrm{e}^{\,-\mathrm{i}\omega t}\,. \tag{5}$$

ω is the angular frequency of light and β denotes the propagation constants.

Maxwell's equations for vanishing charges and currents and for a non-magnetic medium read

$$\boldsymbol{\nabla} \times \boldsymbol{H} = -\mathrm{i}\omega\epsilon_0\epsilon\boldsymbol{E} \quad \text{and} \quad \boldsymbol{\nabla} \times \boldsymbol{E} = \mathrm{i}\omega\mu_0\boldsymbol{H}\,, \tag{6}$$

in usual notation. $\epsilon = \epsilon(x)$ is the relative permittivity. Note that both divergence equations are automatically satisfied for $\omega \neq 0$.

[1] We use the same symbol for a field and the corresponding mode amplitude.

A TE mode is specified by

$$\boldsymbol{E} = \begin{pmatrix} 0 \\ E \\ 0 \end{pmatrix} \quad \text{and} \quad \boldsymbol{H} = \frac{-1}{\omega\mu_0} \begin{pmatrix} \beta E \\ 0 \\ \mathrm{i}E' \end{pmatrix} , \tag{7}$$

a TM mode by

$$\boldsymbol{H} = \begin{pmatrix} 0 \\ H \\ 0 \end{pmatrix} \quad \text{and} \quad \epsilon\,\boldsymbol{E} = \frac{1}{\omega\epsilon_0} \begin{pmatrix} \beta H \\ 0 \\ \mathrm{i}H' \end{pmatrix} . \tag{8}$$

Generally ϵE_x must be continuous as well as E_y, E_z, H_x, H_y, and H_z. In the case of a TE mode, E as well as E' must be continuous functions. For a TM mode, H and H'/ϵ have to be continuous.

TM mode

Generally, the magnetic field strength obeys the second order equation

$$\boldsymbol{\nabla} \times \frac{1}{\epsilon} \boldsymbol{\nabla} \times \boldsymbol{H} = k_0^2 \boldsymbol{H} , \tag{9}$$

with $\epsilon_0\mu_0 c^2 = 1$ and $\omega = k_0 c$. For a TM surface mode this boils down to

$$\left\{ \epsilon(x) \frac{\mathrm{d}}{\mathrm{d}x} \frac{1}{\epsilon(x)} \frac{\mathrm{d}}{\mathrm{d}x} + k_0^2\, \epsilon(x) \right\} H = \beta^2 H . \tag{10}$$

For a piecewise constant permittivity profile, (10) simplifies to

$$H'' + k_0^2\, \epsilon(x)\, H = \beta^2 H . \tag{11}$$

Let us define[2]

$$\kappa_\mathrm{c} = +\sqrt{\beta^2 - k_0^2 \epsilon_\mathrm{c}} . \tag{12}$$

Hence, the solution of (11) in the cover region is

$$H(x) = \mathrm{e}^{-\kappa_\mathrm{c} x} , \tag{13}$$

which vanishes with $x \to \infty$.

We likewise define

$$H(x) = \mathrm{e}^{+\kappa_\mathrm{m} x} \tag{14}$$

within the metall, where

$$\kappa_\mathrm{m} = +\sqrt{\beta^2 - k_0^2 \epsilon_\mathrm{m}} . \tag{15}$$

With our convention for the square root, (14) will vanish with $x \to -\infty$.

Moreover, the magnetic field ist continuous at $x = 0$. In order for $H'(x)/\epsilon(x)$ to be continuous there we require

$$\frac{-\kappa_\mathrm{c}}{\epsilon_\mathrm{c}} = \frac{\kappa_\mathrm{m}}{\epsilon_\mathrm{m}} . \tag{16}$$

[2] We choose the square root of a complex number such that its real part is positive.

Squaring this expression yields

$$\beta^2 = k_0^2 \frac{\epsilon_\mathrm{m}\, \epsilon_\mathrm{c}}{\epsilon_\mathrm{m} + \epsilon_\mathrm{c}} \, . \tag{17}$$

An ideal medium would have a large negative real permittivity. Then, β^2 is positive, and an undamped TM mode will propagate at the interface. For a small imaginary contribution to the metal permittivity, a propagation constant β will result with only a small imaginary contribution. The corresponding TM mode will be weakly damped. Note that there is at most one solution to (16).

For a piecewise constant permittivity the TE mode equation is the same as (11), with H replaced by E. Also the solution is the same. However, this time the field E and its derivative E' must be continuous. Therefore, (16) has to be replaced by

$$-\kappa_\mathrm{c} = \kappa_\mathrm{m} \, . \tag{18}$$

Squaring this implies $\epsilon_\mathrm{c}^2 = \epsilon_\mathrm{m}^2$, hence $\kappa_\mathrm{c} = \kappa_\mathrm{m} = 0$. There is no TE polarized surface mode. This finding holds true for any material on both sides of the interface.

Permittivity for gold

Johnson and Christy have investigated the optical properties of the noble metals[3], among them gold. ϵ_∞ in (4) has the value 9.5. The plasma resonance frequency is described by $\hbar\omega_\mathrm{p} = 8.95$ eV, friction by $\hbar\gamma = 0.069$ eV. We have plotted the corresponding dispersion curve in Figure 1. The free electron gas model fits data well for wavelengths below 2.25 eV.

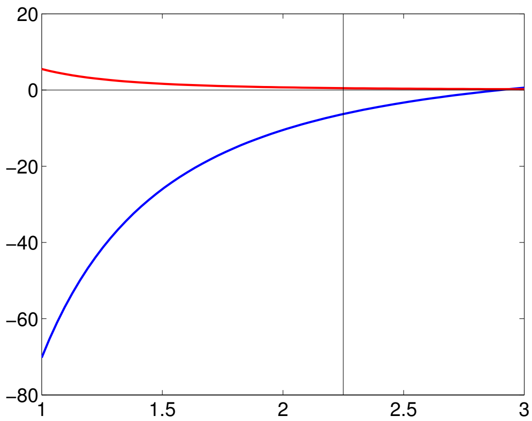

Figure 1: Permittivity of gold according to the free electron gas model. The real part (lower curve) and the imaginary part (upper curve) are plotted versus photon energy in eV. The model fits experimental data well for photon energies below 2.25 eV (vertical line).

[3] P. B. Johnson and R. W. Christy, *Optical constants of the noble metals*, Phys.Rev.B (1972), 4370

An example

We calculate the TM surface mode for $\hbar\omega = 2.25$ eV. The propagation length ℓ is defined by $2\ell \operatorname{Im} \beta = 1$. Data are collected in Table 1.

$\hbar\omega$	2.25	eV
k_0	11.40	μm^{-1}
λ	0.5513	μm
ϵ_c	1.000	
ϵ_m	-6.308+0.4848 i	
β	12.42+0.08925 i	μm^{-1}
κ_c	4.931+0.2247 i	μm^{-1}
κ_m	31.21-0.9730 i	μm^{-1}
ℓ	5.602	μm

Table 1: The TM polarized mode at the surface of gold for light of $\hbar\omega$ = 2.25 eV. See the text for the meaning of the symbols.

The shape of the mode is plotted in Figure 2.

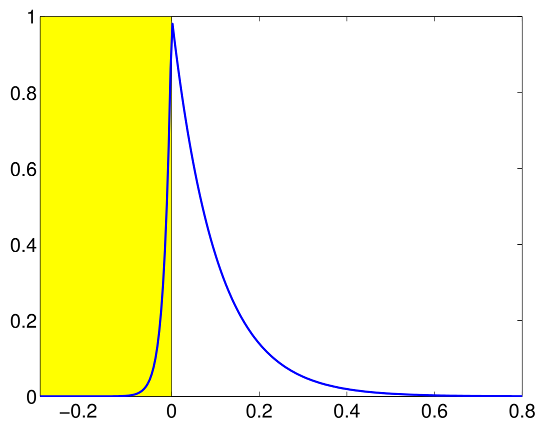

Figure 2: Gold (shaded) is covered by air. The intensity of the TM surface mode corresponding to the above data is plotted vs. the distance (in μm) from the surface. The mode penetrates about 20 nm into the metal.