Bringing Mathematics to Physics

Peter Hertel

Environment Early life Trinity college Calculus Gravity Optics Later life Isaak Newton Bringing Mathematics to Physics

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Gravity

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Later life

Honors an death



Isaak Newton, 1643–1727

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His signature

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- During Newton's lifetime (1643–1727) ...
- England, Scotland, Wales and Northern Ireland were united in personal union by James VI
- In 1707 the United Kingdom (Great Britain) was founded

Environment

- With a constitution and a common parliament
- The Anglican Church was the official church of the country
- The British Navy had won decisive battles over Spain
- ... Britannia rules the waves
- London then the most important city of the world
- Before: zero longitude fixed at Canarian Island El Hierro
- Later: at Royal Observatory at Greenwich (London)
- Outstanding universities at Oxford, Cambridge and Edinburgh



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United Kingdom, as of Newton's later years

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The Spanish armada repeatedly beaten in the Anglo-Spanish war 1585–1604

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Britannia rules the waves.

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Canary islands.

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The Royal Observatory at Greenwich.

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Mathematics and Physics in the UK

- Practical needs often drive scientific progress
- Henry Briggs: 1561-1630
- Arithmetica Logarithmica, 1624
- Tables of the logarithms of sines and tangents, 1631
- Robert Hooke: 1635–1703
- linear relation between stress and strain
- constructional mechanics
- spring-balance watch (Huygens?)
- urgently required for navigation
- Means of transportation were ships and coaches drawn by horses,
- or riding a horse
- Royal Mail since 1516

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- Born at Christmas day 1642 (Julian Calender)
- January 1643 (Gregorian Calender)
- His father, a wealthy farmer, was already dead since three months

Early life

- Born prematurely, he was very tiny
- At the age of three, his mother re-married, and he was cared for by his grand-mother
- Went to school at King's College at Grantham
- Had to leave school with 16 because his newly widowed mother wanted him to become a farmer
- He hated farming,
- Was sent back to school to complete his education as pressured by the headmaster

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• Newton, 18 years old, was admitted to Trinity College at Cambridge

University

- Studying Aristotle, Descartes, Copernicus, Kepler and Galilei
- Works on his theory of infinitesimal calculus
- Degree in 1665
- University closed for two years, precaution against the Great Plague
- Studying at home for the next two years
- Calculus
- Optics
- Gravitation
- Return to Trinity as a fellow (assistant professor)

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Trinity college

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The Great Plague, 1665-1666

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Opticks, printed in 1704, London

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Newtons own copy with corrections

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- There was a bitter dispute on who had invented the differential calculus
- Today, most historians are convinced that Newton and Leibniz developed calculus independently
- However, Leibniz's notation lives up to now

• for
$$f = f(t)$$
 we write
 $\dot{f}(t) = \frac{\mathrm{d}f(t)}{\mathrm{d}t}$

- according to Newton (left) and Leibniz (right)
- Leibniz's infinitesimals $\mathrm{d}f$ make infinitesimal calculus simple, such as

$$(f + \mathrm{d}f)(g + \mathrm{d}g) = fg + g\mathrm{d}f + f\mathrm{d}g + \dots$$

• resulting in $\frac{\mathrm{d}(fg)}{\mathrm{d}x} = g\frac{\mathrm{d}f}{\mathrm{d}x} + f\frac{\mathrm{d}g}{\mathrm{d}x}$

Leibniz or Newton?

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- Both interpreted the derivative as the slope of a tangent
- Both derived the laws for differentiating sums, products and fractions of functions
- The derivative of an inverse function is very easy to work out in Leibniz's notation
- Both knew the fundamental theorem of calculus:

$$f(b) - f(a) = \int_{a}^{b} \mathrm{d}x \, \frac{\mathrm{d}f(x)}{\mathrm{d}x}$$

and interpreted the integral as we do now

- Leibniz published in scholarly Latin, in French and in German
- the integral sign stands for "somme" in French, or sum

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Gottfried Wilhelm Leibniz, 1646-1716

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A calculation device invented by Leibniz.

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• An invention by Newton

- Look for the zeros of a function f = f(x)
- Guess an initial value x_1
- Iterate

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Example $f(x) = x^2 y$
- solution is $x = \sqrt{y}$
- iteration boils down to $x_{n+1} = \frac{x_n^2 y}{x_n}$

$$n+1 = -2x_n$$

•
$$x_0 = 1, y = 2$$

- 1.500000000, 1.4166666667, 1.414215868 ...
- 1.414213562 remains stable (10 digits)

Finding roots by iteration

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- Today it is simple
- Use frame of reference where $\ddot{x} = 0$ if there are no forces (inertial frame)

Gravitation

- Momentum $oldsymbol{p}=m\dot{oldsymbol{x}}$ where m is the mass of the body
- $\dot{\boldsymbol{p}} = \boldsymbol{F}$ where F is a physical force
- The force exerted by a mass M at ${\boldsymbol X}$ on another mass m at ${\boldsymbol x}$ is

$$\boldsymbol{F} = -\frac{GMm(\boldsymbol{x} - \boldsymbol{X})}{|\boldsymbol{x} - \boldsymbol{X}|^3}$$

- attractive $1/r^2$ law, $actio{=}reactio, \ {\rm proportionality}$ to m and M highly plausible
- Newton could explain Keplers's findings.

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- Gravitation is universal: masses attract each other
- The sun the planets
- The planets their moons
- The earth the apples on a tree
- Explanation of Kepler's law
- Spherical mass distribution acts
- as if all the mass were concentrated at its center
- hence

$$g = \frac{GM}{R^2}$$

where \boldsymbol{M} and \boldsymbol{R} are mass and radius of the earth

 $\bullet \ g$ depends only weakly on height

The famous apple

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- 1670–1672 Lectures on optics
- Decomposition of white light into colors by a prism
- Composition of the colors into white light by another prism or a lense
- Color is a property of light, not of objects
- Refractor telescope suffers from dispersion
- Reflector telescope (Newton's telescope)
- Demonstration to the Royal Society
- Newton's rings



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Newton discovered that white light is composed of colored light.

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Telescope shown to the Royal Society (replica)

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Largest telescope GTC as of 2010 at La Palma, Canary islands. The mirror has a diameter of 10.4 m. It consists of 36 segments which are adjusted individually.

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Newton's rings

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- Religious tracts
- Alchemy
- Elected to represent Cambridge University in Parliament of England (1689–90, 1701)

Later life

- mentioned only once in the records ...
- demanding to close the window because of draught
- was appointed as Master of the Mint (1696)
- Although a *sinecure* (paid job without duty)
- Newton took the job serious, retiring from Cambridge and moving to London
- fighting counterfeiting
- changed the pound Sterling from silver to gold standard

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TAke reddifh rich Virgin Earth in Υ , impregnate it with \bigcirc_{2} , berene and dew, little he end of May: Then imbibe fprinklingly with dew gathered in May, and dry in \bigcirc_{2} expose all Night to the 2 and Air, fecuring it from Rain. Still when it is dry, imbibe and turn the Earth often. Continue this till emation. The hot \bigcirc (effecially in the Dog-days) will make a pure Salt fhootup, which mingle back into the Earth, by turning it all over. Then diffill by graduated \triangle as A. P. forting all the Spirits

Alchemy recipies

An Explication of the Characters which are used in this Book.

Gold.	A.F. Aqua Fortis.
Silver.	A. R. Aqua Regu.
f Iron.	S.V. Spirit of Wine
Mercury.	🔉 Sublimate,
4 Jupiter.	🖙 Precipitata.
9. Venus.	ana Amalgama.
h Lead.	V Water.
8 Antimony.	A Fing.
Sal armoniac.	

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British Parliament

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The Tower of London, also seat of the mint.

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- Was elected President of the Royal Society, 1703–1727
- Associated member of the French Académie des Sciences

Honors

- Queen Anne knighted him in 1705
- probably not for his merits, but for political reasons
- shortly before parliamentary elections
- Sir Isaak Newton was the second scientist (after Sir Francis Bacon)
- Died in sleep, 1727
- Considerable amount of mercury found in his hair (alchemy?)
- Buried at Westminster Abbey
- Later Charles Darwin was buried there close by

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Sir Isaak Newton, coat of arms.

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Westminser Abbey

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Newton's tomb in Westminster Abbey

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Later life

Honors and death I could see further because I was standing on the shoulders of giants.

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Galileo Galilei, 1564-1642

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Isaak Newton, 1643–1727