Planar Waveguides

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Modes

- permittivity $\epsilon = \epsilon(x)$ is constant in x, y plane
- wave vector is perpendicular to x axis, we choose z
- all fields are of the form $F(t, x, y, z) = F(x)\,\mathrm{e}^{\mathrm{i}\beta z}\,\mathrm{e}^{-\mathrm{i}\omega t}$
- β is the propagation constant of the mode
- there are two kind of modes:
- TE - Transversal Electric
- TM - Transversal Magnetic
- they have different propagation constants
- $\epsilon E_x, E_y, E_z, H_x, H_y, H_z$ must be continuous

TE modes

- $\boldsymbol{E} = (0, E, 0)$
- $\mathrm{i}\omega\mu_0 \boldsymbol{H} = \mathbf{curl}\, \boldsymbol{E}$
- $\boldsymbol{H} = (-\beta E, 0, -\mathrm{i}E')/\omega\mu_0$
- The TE mode equation reads
  $$\frac{1}{k_0^2} E'' + \epsilon(x) E = \epsilon_{\text{eff}} E$$
- $n_{\text{eff}} = \beta/k_0$ effective index
- $\epsilon_{\text{eff}} = n_{\text{eff}}^2$ effective permittivity
- eigenvalue problem, with boundary condition
- $E(x) \to 0$ for $x \to \pm\infty$
- E and E' must be continuous functions

TM modes

- $\boldsymbol{H} = (0, H, 0)$
- $-\mathrm{i}\omega\epsilon_0\,\epsilon(x)\,\boldsymbol{E} = \mathbf{curl}\,\boldsymbol{H}$
- $\boldsymbol{E} = (\beta H, 0, \mathrm{i}H')/\omega\epsilon_0\epsilon(x)$
- The TM mode equation reads
$$\frac{1}{k_0^2}\,\epsilon(x)\,\frac{\mathrm{d}}{\mathrm{d}x}\,\epsilon(x)^{-1}\,\frac{\mathrm{d}}{\mathrm{d}x}H + \epsilon(x)\,H = \epsilon_{\mathrm{eff}}H$$
- recall $n_{\mathrm{eff}} = \beta/k_0$ effective index
- $\epsilon_{\mathrm{eff}} = n_{\mathrm{eff}}^2$ effective permittivity
- eigenvalue problem, with boundary condition
- $H(x) \to 0$ for $x \to \pm\infty$
- H and H'/ϵ must be continuous functions
- recall $\epsilon E_x, E_y, E_z, H_x, H_y, H_z$ must be continuous

Graded index waveguide

- Cover (air) and Substrate (glass, lithium niobate)
- increased permittivity beneath surface
- indiffusion (of Ti in lithium niobate)
- field assisted ion exchange (silver in glass)
- permittivity profile

$$\epsilon(x) = \begin{cases} \epsilon_{\rm c} \text{ for } x < 0 \\ \epsilon_{\rm s} + \Delta\epsilon\, {\rm e}^{-(x/W)^2} \text{ for } x > 0 \end{cases}$$

- gaussian diffusion profile

```
LAMBDA=0.6328;
k0=2*pi/LAMBDA;
EC=1.000;
ES=4.800;
ED=0.045;
W=4.00;
xmin=-1.0;
xmax=4*W;
h=0.1;
x=(xmin:h:xmax)';
dim=size(x,1);
prm=(x<0).*EC+(x>=0).*(ES+ED*exp(-(x/W).^2));
next=ones(dim-1,1)/h^2/k0^2;
main=-2.0*ones(dim,1)/h^2/k0^2+prm;
L=diag(next,-1)+diag(main,0)+diag(next,1);
[evec, eval]=eig(L);
eff_eps=diag(eval);
guided=evec(:,eff_eps>ES);
plot(x,guided);
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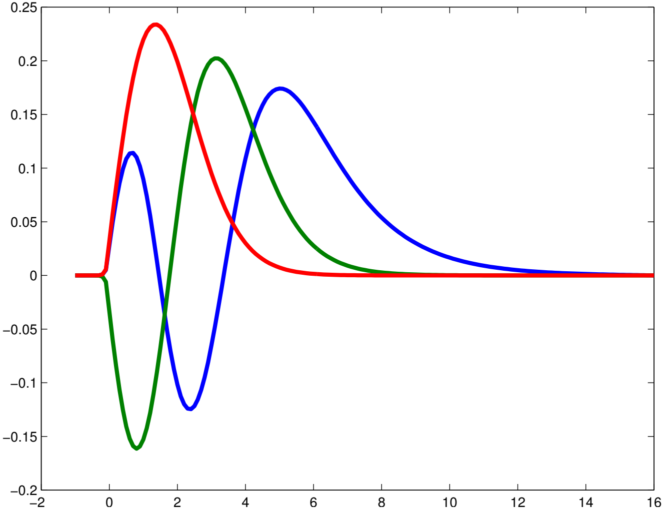
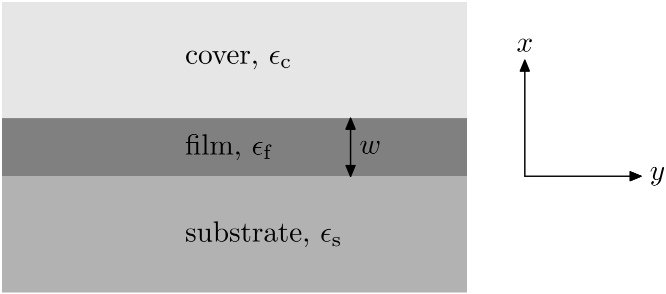


The guided modes of a graded index waveguide.

Slab waveguides

- A slabwaveguide consists of a substrate
- one or more films of enhanced permittivities
- and is covered by a layer of low permittivity
- the permittivity profile

$$\epsilon(x) = \begin{cases} \epsilon_{\rm s} \text{ for } x < 0 \\ \epsilon_{\rm f} \text{ for } 0 < x < w \\ \epsilon_{\rm c} \text{ for } w < x \end{cases}$$



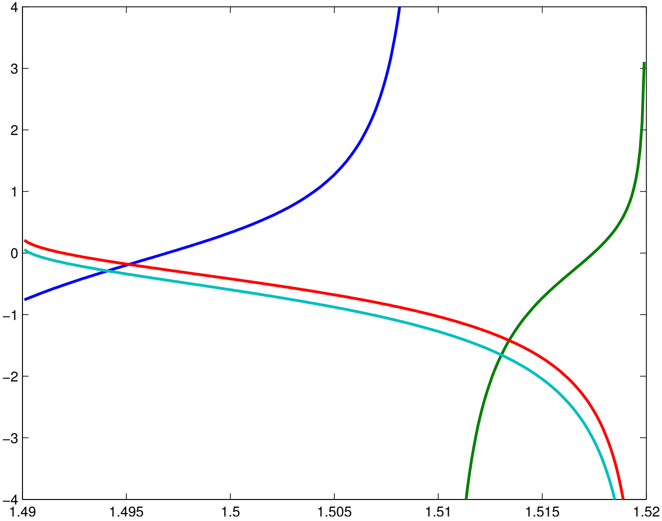
A slab waveguide. A film of increased permittivity on top of a substrate being covered by material of low permittivity. w is the film thickness.

Analytical solution I

- substrate:
  $E = \mathrm{e}^{\kappa_\mathrm{s} x}$ where $\kappa_\mathrm{s} = k_0\sqrt{\epsilon_\mathrm{eff} - \epsilon_\mathrm{s}}$
- film:
  $E = c\cos k_\mathrm{f}\, x + s\sin k_\mathrm{f}\, x$ where $k_\mathrm{f} = k_0\sqrt{\epsilon_\mathrm{f} - \epsilon_\mathrm{eff}}$
- E and E' continuous at $x = 0$
- $1 = c$ and $\kappa_\mathrm{s} = s\, k_\mathrm{f}$
- cover:
  $E = a\,\mathrm{e}^{-\kappa_\mathrm{c} x} + b\,\mathrm{e}^{+\kappa_\mathrm{c} x}$ where $\kappa_\mathrm{c} = k_0\sqrt{\epsilon_\mathrm{eff} - \epsilon_\mathrm{c}}$
- E and E' continuous at $x = w$, b must vanish

Analytical solution II

- $b = 0$ for TE modes results in
$$\cot k_\mathrm{f} w = \frac{k_\mathrm{f}^2 - \kappa_\mathrm{s}\kappa_\mathrm{c}}{k_\mathrm{f}(\kappa_\mathrm{s} + \kappa_\mathrm{c})}$$
- for TM modes: H and H'/ϵ must be continuous
- which results in
$$\cot \bar{k}_\mathrm{f} w = \frac{\bar{k}_\mathrm{f}^2 - \bar{\kappa}_\mathrm{s}\bar{\kappa}_\mathrm{c}}{\bar{k}_\mathrm{f}(\bar{\kappa}_\mathrm{s} + \bar{\kappa}_\mathrm{c})}$$
- $\bar{\kappa}_\mathrm{c} = \kappa_\mathrm{c}/\epsilon_\mathrm{c}$, $\bar{k}_\mathrm{f} = k_\mathrm{f}/\epsilon_\mathrm{f}$, $\bar{\kappa}_\mathrm{s} = \kappa_\mathrm{s}/\epsilon_\mathrm{s}$
- example: n_f=1.52, w_f=1.8 μm, n_s=1.49, n_c=1.00, λ=632.8 nm.



Graphical solution for TE and TM mode conditions. Blue and green are the branches of the cotangent if plotted versus the effective index. Red and teal represent the right hand sides for TE and TM modes, respectively.

Java applet

http://wwwhome.math.utwente.nl/~hammerm/oms.html