Properties of susceptibility

Peter Hertel

University of Osnabrück, Germany

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Recall

- $P_i(t, \mathbf{x}) = \sum_j \int d\tau \int d^3\boldsymbol{\xi}\ \theta(\tau)\Gamma_{ij}u, \boldsymbol{\xi})E_j(t-\tau, \mathbf{x}-\boldsymbol{\xi})$

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- We shall study next

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Kramers-Kronig relations

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- χ'' describes absorption. Why must it be positive?

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- $K(A,B,0)$ describes the correlation between A and B

Wiener-Khintchin theorem (ctd)

- Let us insert Fourier-transforms

$$A(t+\tau) = \langle A \rangle + \int \frac{d\omega}{2\pi}\, e^{-i\omega(t+\tau)}\, \hat{A}(\omega)$$

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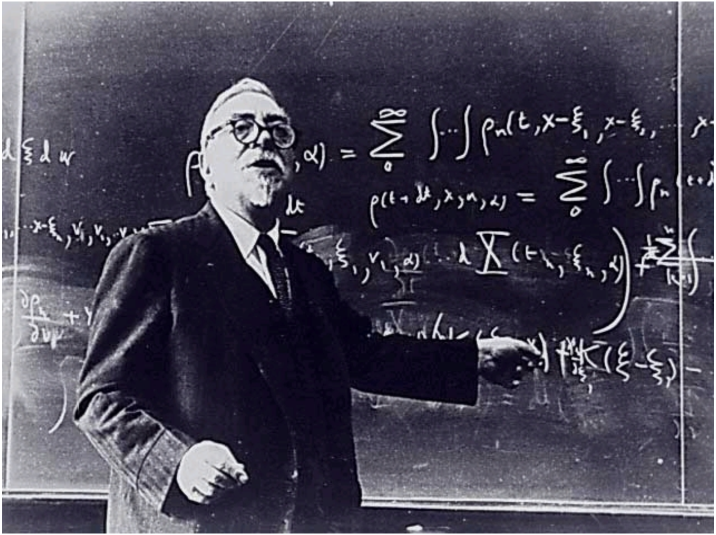
- The Wiener Khinchin theorem says

$$K(AA;\tau) = \int \frac{d\omega}{2\pi}\, e^{-i\omega\tau}\, S(A,A;\omega)$$

with $S(AA;\omega) \geq 0$



Norbert Wiener

Aleksandr Yakovlevich Khinchin

Kubo-Martin-Schwinger formula

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- where $\beta = 1/kBT$

Ryogo Kubo

Paul Martin

Julian Schwinger

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- $< BA(\tau) > - < A >< B > = f(\tau - i\hbar\beta)$

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- particularly interesting for $A = B$

J. Callen