Perturbing the Equilibrium

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- if m_i was found, system is in (pure) state ϕ_i

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- Lagrange parameters F and T are free energy and temperature, respectively

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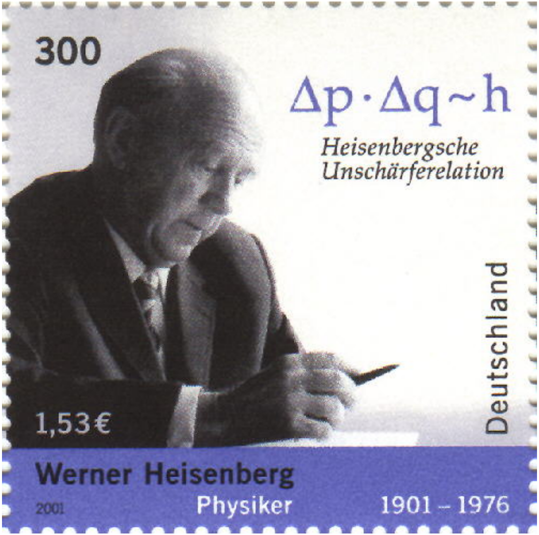
- energy is conserved because $\frac{d}{dt} H_t = 0$



Werner Heisenberg

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- We shall work out the linear response of the system to small perturbations

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