

# Metamaterials

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- Metamaterials are produced artificially
- with strange optical properties
- for instance negative dielectric permittivity
- \*and\* negative magnetic permeability
- split ring resonator SRR
- Snellius law and more
- Same for a backward material
- Applications

# Metamaterials

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## Overview

Optical properties of normal matter

Artificial resonators

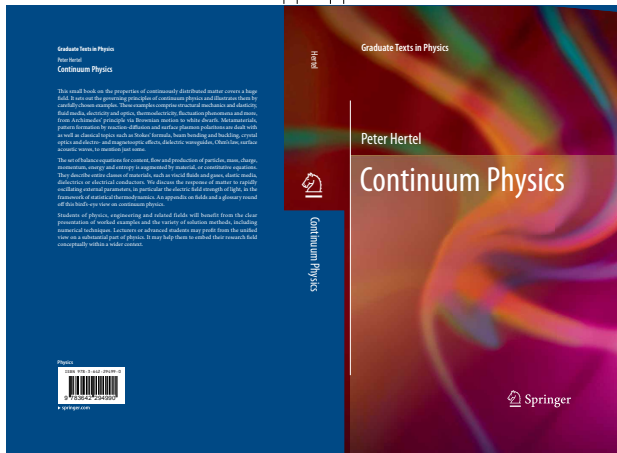
Optics

Normal refraction

Refraction in a backward medium

An application

Summary



Will appear soon

- harmonically oscillating field

$$\mathbf{E}(t) = \tilde{\mathbf{E}} e^{-i\omega t}$$

- elastically bound damped electron

$$m\{\ddot{\mathbf{x}} + \Gamma\dot{\mathbf{x}} + \Omega^2\mathbf{x}\} = q\mathbf{E}$$

- Fourier transform it

$$\tilde{\mathbf{x}} = \frac{q}{m} \frac{\tilde{\mathbf{E}}}{-\omega^2 - i\Gamma\omega + \Omega^2}$$

- polarization for  $N$  electrons per unit volume

$$\tilde{\mathbf{P}} = \frac{Nq^2}{m} \frac{\tilde{\mathbf{E}}}{-\omega^2 - i\Gamma\omega + \Omega^2}$$

- magnetization  $\tilde{\mathbf{M}}$  negligibly small

- susceptibility  $\chi$  defined by

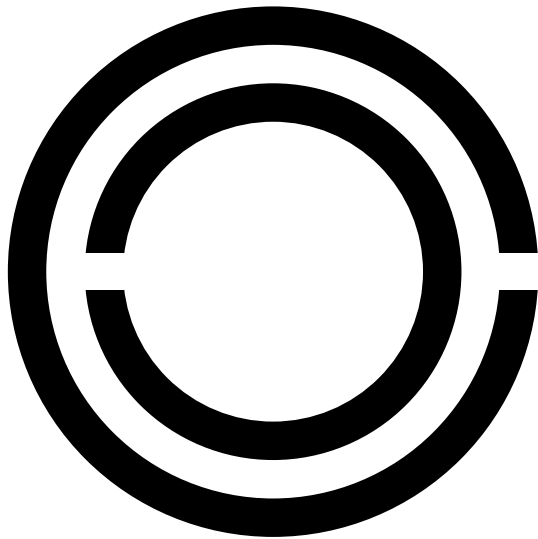
$$\tilde{\mathbf{P}} = \chi(\omega)\epsilon_0\tilde{\mathbf{E}}$$

- generalized Drude model

$$\chi(\omega) = \sum_a \frac{f_a \Omega_a^2}{-\omega^2 - i\Gamma_a \omega + \Omega_a^2}$$

- sum over all resonance frequencies  $\Omega_a$  with weights  $f_a$  (oscillator strength)
  - a variant of this is also known as Sellmeier's formula
  - permittivity function  $\epsilon(\omega)$  defined by
- $$\tilde{\mathbf{D}} = \epsilon(\omega)\epsilon_0\tilde{\mathbf{E}}$$
- $\epsilon(\omega) = 1 + \chi(\omega)$
  - permittivity of natural materials, like solids or liquids

- If you want strange optical effects ...
- you must provide for strange resonators,
- not just elastically bound damped electrons.
- Metamaterials are arrays of resonating circuits
- the dimension of which is small if compared with the wavelength
- can be realized easily for microwaves
- Optical metamaterials require advanced nano-technology
- such as self-assembling
- as of today: science fiction



The split ring resonator. Inductance  $L$  and capacitance  $C$ .  
Resonance frequency is  $\Omega = 1/\sqrt{LC}$ .

- Maxwell's equations for plain waves

$$f(t, \mathbf{x}) = \tilde{f} e^{-i\omega t} e^{i\mathbf{k} \cdot \mathbf{x}}$$

- curl of the magnetic field

$$\mathbf{k} \times \tilde{\mathbf{H}} = -\omega\epsilon(\omega)\epsilon_0\tilde{\mathbf{E}}$$

- curl of the electric field

$$\mathbf{k} \times \tilde{\mathbf{E}} = \omega\mu(\omega)\mu_0\tilde{\mathbf{H}}$$

- $\mathbf{k} = k\hat{\mathbf{k}}$  and  $\tilde{\mathbf{E}} = \tilde{E}\hat{\mathbf{e}}$

- wave number, propagation direction, amplitude, polarization vector

- solution

$$\tilde{\mathbf{H}} = \frac{k\tilde{E}}{\omega\mu\mu_0}\hat{\mathbf{k}} \times \hat{\mathbf{e}}$$



- dispersion relation

$$k^2 c^2 = \omega^2 \epsilon(\omega) \mu(\omega)$$

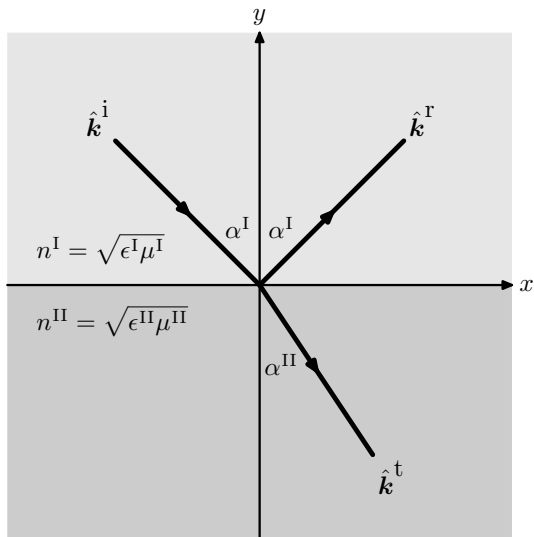
- or, with  $k_0 = \omega/c$

$$k = n(\omega) k_0 \text{ where } n(\omega) = +\sqrt{\epsilon(\omega)\mu(\omega)}$$

- Poynting vector  $\mathbf{S} = 2 \operatorname{Re} \mathbf{E}^* \times \mathbf{H}$

$$\mathbf{S} = \frac{2n}{c\mu\mu_0} |\tilde{E}|^2 \hat{\mathbf{k}}$$

- $n$  real if  $\epsilon > 0$  and  $\mu > 0$
- this is a **forward** medium
- $n$  also real if  $\epsilon < 0$  and  $\mu < 0$
- **backward** medium
- *meta instead of natural material*



Incident, transmitted and reflected beam at interface of normal materials

- let us study perpendicularly polarized light
- only  $E = E_z$  components do not vanish
- $y > 0$ : incident+reflected;  $y < 0$ : transmitted

$$E^i = E^i e^{ik_0 n^I (\sin \alpha^i x - \cos \alpha^i y)}$$

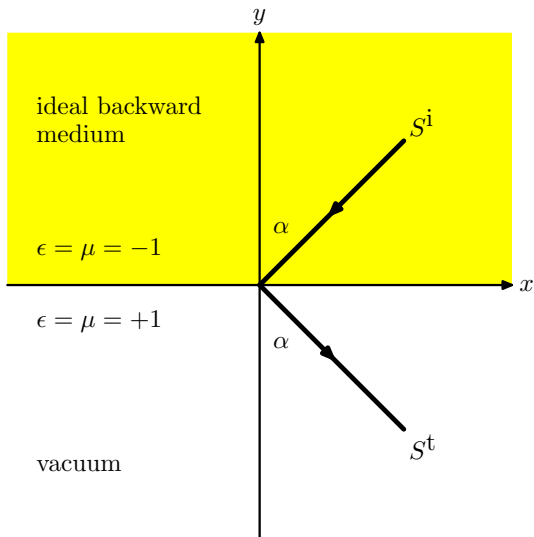
$$E^r = E^r e^{ik_0 n^I (\sin \alpha^r x + \cos \alpha^r y)}$$

$$E^t = E^t e^{ik_0 n^{II} (\sin \alpha^t x - \cos \alpha^t y)}$$

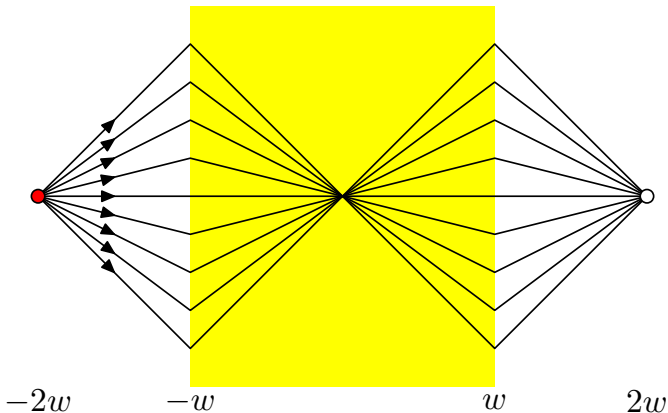
- tangential components must be continuous
- $E^i + E^r = E^t$
- incident and reflected beam angles are equal, Snell's law  
 $n^I \sin \alpha^i = n^I \sin \alpha^r = n^{II} \sin \alpha^t$
- $H_x, H_z$  and  $\mu H_y$  must be continuous
- satisfied if

$$\frac{n^I \cos \alpha^I}{\mu^I} (E^i - E^r) = \frac{n^{II} \cos \alpha^{II}}{\mu^{II}} E^t$$

- we discuss an **ideal backward medium**
- $\epsilon = -1$  and  $\mu = -1$
- and its plane interface with vacuum
- refractive indexes of both media are  $n^{\text{I}} = n^{\text{II}} = 1$
- all angles are equal
- as before:  $E^{\text{r}} + E^{\text{i}} = E^{\text{t}}$
- different:  $E^{\text{r}} - E^{\text{i}} = E^{\text{t}}$
- therefore  $E^{\text{i}} = 0$  and  $E^{\text{r}} = E^{\text{t}}$
- the formerly reflected beam is now incident
- the formerly incident beam vanishes



Power flow of a a beam which passes from an ideal backward medium to vacuum.



A parallel slab of ideal backward material. It maps the upward object at the left into an equally large upward image at the right. No diffraction limitation!

- Metamaterials consist of regularly arranged LC resonators
- Realized for microwaves, still science fiction for optics
- progress in nano technology expected
- in particular, self-assembling
- metamaterials show positive and negative permittivities and permeabilities
- if both are negative - in a certain frequency range - the energy flows counter to wave propagation
- metamaterials may be backward
- strange behavior at interfaces between normal and metamaterial
- example: a new microscope not restricted by the diffraction limit